

GOVT POLYTECHNIC KENDRAPARA



LECTURES NOTE

Digital Electronics & Microprocessor

5TH SEM ELECTRICAL

PREPARED BY

NIGAM PRASAD

MOHAPATRA

PTGF ELECTRICAL

UNIT - 1

NUMBER SYSTEMS & BOOLEAN ALGEBRA

- Introduction about digital system
- Philosophy of number systems
- Complement representation of negative numbers
- Binary arithmetic
- Binary codes
- Error detecting & error correcting codes
- Hamming codes

INTRODUCTION ABOUT DIGITAL SYSTEM

A Digital system is an interconnection of digital modules and it is a system that manipulates discrete elements of information that is represented internally in the binary form.

Now a day's digital systems are used in wide variety of industrial and consumer products such as automated industrial machinery, pocket calculators, microprocessors, digital computers, digital watches, TV games and signal processing and so on.

Characteristics of Digital systems

- Digital systems manipulate discrete elements of information.
- Discrete elements are nothing but the digits such as 10 decimal digits or 26 letters of alphabets and so on.
- Digital systems use physical quantities called signals to represent discrete elements.
- In digital systems, the signals have two discrete values and are therefore said to be binary.
- A signal in digital system represents one binary digit called a bit. The bit has a value either 0 or 1.

Analog systems vs Digital systems

Analog system process information that varies continuously i.e; they process time varying signals that can take on any values across a continuous range of voltage, current or any physical parameter.

Digital systems use digital circuits that can process digital signals which can take either 0 or 1 for binary system.

Advantages of Digital system over Analog system

1. Ease of programmability

The digital systems can be used for different applications by simply changing the program without additional changes in hardware.

2. Reduction in cost of hardware

The cost of hardware gets reduced by use of digital components and this has been possible due to advances in IC technology. With ICs the number of components that can be placed in a given area of Silicon are increased which helps in cost reduction.

3. High speed

Digital processing of data ensures high speed of operation which is possible due to advances in Digital Signal Processing.

4. High Reliability

Digital systems are highly reliable one of the reasons for that is use of error correction codes.

5. Design is easy

The design of digital systems which require use of Boolean algebra and other digital techniques is easier compared to analog designing.

6. Result can be reproduced easily

Since the output of digital systems unlike analog systems is independent of temperature, noise, humidity and other characteristics of components the reproducibility of results is higher in digital systems than in analog systems.

Disadvantages of Digital Systems

- Use more energy than analog circuits to accomplish the same tasks, thus producing more heat as well.
- Digital circuits are often fragile, in that if a single piece of digital data is lost or misinterpreted the meaning of large blocks of related data can completely change.
- Digital computer manipulates discrete elements of information by means of a binary code.
- Quantization error during analog signal sampling.

NUMBER SYSTEM

Number system is a basis for counting various items. Modern computers communicate and operate with binary numbers which use only the digits 0 & 1. Basic number system used by humans is Decimal number system.

For Ex: Let us consider decimal number 18. This number is represented in binary as 10010.

We observe that binary number system take more digits to represent the decimal number. For large numbers we have to deal with very large binary strings. So this fact gave rise to three new number systems.

- i) Octal number systems
- ii) Hexa Decimal number system
- iii) Binary Coded Decimal number(BCD) system

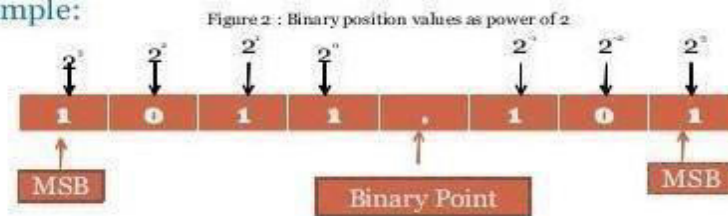
To define any number system we have to specify

- Base of the number system such as 2,8,10 or 16.
- The base decides the total number of digits available in that number system.
- First digit in the number system is always zero and last digit in the number system is always base-1.

Binary number system:

The binary number has a radix of 2. As $r = 2$, only two digits are needed, and these are 0 and 1. In binary system weight is expressed as power of 2.

• **Example:**



The left most bit, which has the greatest weight is called the Most Significant Bit (MSB). And the right most bit which has the least weight is called Least Significant Bit (LSB).

For Ex: $1001.01_2 = [(1) \times 2^3] + [(0) \times 2^2] + [(0) \times 2^1] + [(1) \times 2^0] + [(0) \times 2^{-1}] + [(1) \times 2^{-2}]$

$$1001.01_2 = [1 \times 8] + [0 \times 4] + [0 \times 2] + [1 \times 1] + [0 \times 0.5] + [1 \times 0.25]$$

$$1001.01_2 = 9.25_{10}$$

Decimal Number system

The decimal system has ten symbols: 0,1,2,3,4,5,6,7,8,9. In other words, it has a base of 10.

Octal Number System

Digital systems operate only on binary numbers. Since binary numbers are often very long, two shorthand notations, octal and hexadecimal, are used for representing large binary numbers. Octal systems use a base or radix of 8. It uses first eight digits of decimal number system. Thus it has digits from 0 to 7.

Hexa Decimal Number System

The hexadecimal numbering system has a base of 16. There are 16 symbols. The decimal digits 0 to 9 are used as the first ten digits as in the decimal system, followed by the letters A, B, C, D, E and F, which represent the values 10, 11,12,13,14 and 15 respectively.

Decima l	Binar y	Octal	Hexadeci mal
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Number Base conversions

The human beings use decimal number system while computer uses binary number system. Therefore it is necessary to convert decimal number system into its equivalent binary.

- i) Binary to octal number conversion
- ii) Binary to hexa decimal number conversion

The binary number: 001 010 011 000 100 101 110 111
 The octal number: 1 2 3 0 4 5 6 7

The binary number: 0001 0010 0100 1000 1001 1010 1101 1111
 The hexadecimal number: 1 2 5 8 9 A D F

- iii) Octal to binary Conversion

Each octal number converts to 3 binary digits

Code
0 - 000
1 - 001
2 - 010
3 - 011
4 - 100
5 - 101
6 - 110
7 - 111

To convert 653_8 to binary, just substitute code:

6 5 3
 ↓ ↓ ↓
 110 101 011

- iv) Hexa to binary conversion
- 0100 1111 1101 0111**

- v) Octal to Decimal conversion

Ex: convert 4057.06_8 to octal

$$=4 \times 8^3 + 0 \times 8^2 + 5 \times 8^1 + 7 \times 8^0 + 0 \times 8^{-1} + 6 \times 8^{-2}$$

$$=2048 + 0 + 40 + 7 + 0 + 0.0937$$

$$=2095.0937_{10}$$

- vi) Decimal to Octal Conversion

Ex: convert 378.93_{10} to octal

378₁₀ to octal: Successive division:

$$\begin{array}{r} 8 \mid 378 \\ \hline 8 \mid 47 \text{ --- } 2 \\ \hline 8 \mid 5 \text{ --- } 7 \quad \uparrow \\ \hline 0 \text{ --- } 5 \end{array}$$

$$=572_8$$

0.93₁₀ to octal :

$$\begin{array}{r} 0.93 \times 8 = 7.44 \\ 0.44 \times 8 = 3.52 \quad \downarrow \\ 0.53 \times 8 = 4.16 \\ 0.16 \times 8 = 1.28 \\ =0.7341_8 \end{array}$$

$$378.93_{10} = 572.7341_8$$

vii) Hexadecimal to Decimal Conversion

Ex: $5C7_{16}$ to decimal

$$\begin{aligned} &= (5 \times 16^2) + (C \times 16^1) + (7 \times 16^0) \\ &= 1280 + 192 + 7 \end{aligned}$$

$$= 147_{10}$$

viii) Decimal to Hexadecimal Conversion

Ex: 2598.67510

$$\begin{array}{r} 16 \overline{) 2598} \\ \underline{162} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

$$= A26_{(16)}$$



$$\begin{aligned}
0.67510 &= 0.675 \times 16 \rightarrow 10.8 \\
&= 0.800 \times 16 \rightarrow 12.8 \quad \downarrow \\
&= 0.800 \times 16 \rightarrow 12.8 \\
&= 0.800 \times 16 \rightarrow 12.8 \\
&= 0.ACCC_{16}
\end{aligned}$$

$$2598.675_{10} = A26.ACCC_{16}$$

ix) Octal to hexadecimal conversion:

The simplest way is to first convert the given octal no. to binary & then the binary no. to hexadecimal.

Ex: 756.603_8

7	5	6	.	6	0	3
111	101	110	.	110	000	011
0001	1110	1110	.	1100	0001	1000
1	E	E	.	C	1	8

x) Hexadecimal to octal conversion:

First convert the given hexadecimal no. to binary & then the binary no. to octal.

Ex: $B9F.AE_{16}$

B	9	F	.	A	E		
1011	1001	1111	.	1010	1110		
101	110	011	111	.	101	011	100
5	6	3	7	.	5	3	4

$$= 5637.534$$

Complements:

In digital computers to simplify the subtraction operation & for logical manipulation complements are used. There are two types of complements used in each radix system.

- i) The radix complement or r 's complement
- ii) The diminished radix complement or $(r-1)$'s complement

Special case in 2's comp representation:

Whenever a signed no. has a 1 in the sign bit & all 0's for the magnitude bits, the decimal equivalent is -2^n , where n is the no of bits in the magnitude .

Ex: 1000= -8 & 10000=-16

Characteristics of 2's compliment no.s:

Properties:

1. There is one unique zero
2. 2's comp of 0 is 0
3. The leftmost bit can't be used to express a quantity . it is a 0 no. is +ve.
4. For an n-bit word which includes the sign bit there are $(2^{n-1}-1)$ +ve integers, 2^{n-1} -ve integers & one 0 , for a total of 2^n unique states.
5. Significant information is contained in the 1's of the +ve no.s & 0's of the -ve no.s
6. A -ve no. may be converted into a +ve no. by finding its 2's comp.

Signed binary numbers:

Decimal	Sign 2's comp form	Sign 1's comp form	Sign mag form
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0011	0011	0011
+0	0000	0000	0000

-0	--	1111	1000
-1	1111	1110	1001
-2	1110	1101	1010
-3	1101	1100	1011
-4	1100	1011	1100
-5	1011	1010	1101
-6	1010	1001	1110
-7	1001	1000	1111
8	1000	--	--

Methods of obtaining 2's comp of a no:

- In 3 ways
 1. By obtaining the 1's comp of the given no. (by changing all 0's to 1's & 1's to 0's) & then adding 1.
 2. By subtracting the given n bit no N from 2^n
 3. Starting at the LSB , copying down each bit upto & including the first 1 bit encountered , and complimenting the remaining bits.
- Ex: Express -45 in 8 bit 2's comp form

+45 in 8 bit form is 00101101

I method:

1's comp of 00101101 & the add 1

```

00101101
11010010
      +1
  
```

11010011 is 2's comp form

II method:

Subtract the given no. N from 2^n

```

      2^n = 100000000
Subtract 45= -00101101
              +1
  
```

11010011 is 2's comp

III method:

Original no: 00101101

Copy up to First 1 bit 1

Compliment remaining : 1101001

bits 11010011

Ex:

-73.75 in 12 bit 2's comp form

I method

$$\begin{array}{r}
01001001.1100 \\
10110110.0011 \\
+1 \\
\hline
\end{array}$$

10110110.0100 is 2's

II method:

$2^8 = 100000000.0000$

Sub 73.75 = -01001001.1100

$$\begin{array}{r}
\hline
10110110.0100 \text{ is 2's comp}
\end{array}$$

III method :

Original no : 01001001.1100

Copy up to 1'st bit 100

Comp the remaining bits: 10110110.0

$$\begin{array}{r}
\hline
10110110.0100
\end{array}$$

2's compliment Arithmetic:

- The 2's comp system is used to rep -ve no.s using modulus arithmetic . The word length of a computer is fixed. i.e, if a 4 bit no. is added to another 4 bit no . the result will be only of 4 bits. Carry if any , from the fourth bit will overflow called the Modulus arithmetic.

Ex: 1100+1111=1011

- In the 2's compl subtraction, add the 2's comp of the subtrahend to the minuend . If there is a carry out , ignore it , look at the sign bit I.e, MSB of the sum term .If the MSB is a 0, the result is positive.& it is in true binary form. If the MSB is a 1 (carry in or no carry at all) the result is negative.& is in its 2's comp form. Take its 2's comp to find its magnitude in binary.

Ex: Subtract 14 from 46 using 8 bit 2's comp arithmetic:

$$\begin{array}{r}
+14 = 00001110 \\
-14 = 11110010 \quad \text{2's comp} \\
+46 = 00101110 \\
-14 = +11110010 \quad \text{2's comp form of -14} \\
\hline
\hline
\end{array}$$

$$\begin{array}{r} \underline{-32} \quad \underline{(1)00100000} \end{array} \quad \text{ignore carry}$$

Ignore carry, The MSB is 0. so the result is +ve. & is in normal binary form. So the result is +00100000=+32.

EX: Add -75 to +26 using 8 bit 2's comp arithmetic

$$\begin{array}{r} +75 = 01001011 \\ -75 = 10110101 \quad \text{2's comp} \\ \hline +26 = 00011010 \\ -75 = +10110101 \quad \text{2's comp form of -75} \\ \hline \underline{-49} \quad \underline{11001111} \quad \text{No carry} \end{array}$$

No carry, MSB is a 1, result is -ve & is in 2's comp. The magnitude is 2's comp of 11001111. i.e, 00110001 = 49. so result is -49

Ex: add -45.75 to +87.5 using 12 bit arithmetic

$$\begin{array}{r} +87.5 = 01010111.1000 \\ -45.75 = +11010010.0100 \\ \hline \underline{-41.75} \quad \underline{(1)00101001.1100} \quad \text{ignore carry} \\ \text{MSB is 0, result is +ve.} = +41.75 \end{array}$$

1's compliment of n number:

- It is obtained by simply complimenting each bit of the no., & also, 1's comp of a no, is subtracting each bit of the no. from 1. This complemented value rep the -ve of the original no. One of the difficulties of using 1's comp is its rep of zero. Both 00000000 & its 1's comp 11111111 rep zero.
- The 00000000 called +ve zero & 11111111 called -ve zero.

Ex: -99 & -77.25 in 8 bit 1's comp

$$\begin{array}{r} +99 = 01100011 \\ -99 = 10011100 \end{array}$$

$$\begin{array}{r} +77.25 = 01001101.0100 \\ -77.25 = 10110010.1011 \end{array}$$

1's compliment arithmetic:

In 1's comp subtraction, add the 1's comp of the subtrahend to the minuend. If there is a carryout, bring the carry around & add it to the LSB called the **end around carry**. Look at the sign bit (MSB). If this is a 0, the result is +ve & is in true binary. If the MSB is a 1 (carry or no carry), the result is -ve & is in its 1's comp form. Take its 1's comp to get the magnitude in binary.

Ex: Subtract 14 from 25 using 8 bit 1's EX: ADD -25 to +14

$$\begin{array}{r}
 25 = 00011001 \\
 -45 = 11110001 \\
 \hline
 +11 \quad (1)00001010 \\
 \hline
 +1 \\
 \hline
 00001011
 \end{array}
 \qquad
 \begin{array}{r}
 +14 = 00001110 \\
 -25 = +11100110 \\
 \hline
 -11 \quad 11110100 \\
 \hline
 \text{No carry MSB}=1 \\
 \text{result}=-\text{ve}=-11_{10}
 \end{array}$$

MSB is a 0 so result is +ve (binary)

=+11₁₀

Binary codes

Binary codes are codes which are represented in binary system with modification from the original ones.

- Weighted Binary codes
- Non Weighted Codes

Weighted binary codes are those which obey the positional weighting principles, each position of the number represents a specific weight. The binary counting sequence is an example.

Decimal	BCD 8421	Excess-3	84-2-1	2421	5211	Bi-Quinary 5043210			5	0	4	3	2	1	0
0	0000	0011	0000	0000	0000	0100001		0	X						X
1	0001	0100	0111	0001	0001	0100010		1	X					X	
2	0010	0101	0110	0010	0011	0100100		2	X			X			
3	0011	0110	0101	0011	0101	0101000		3	X		X				
4	0100	0111	0100	0100	0111	0110000		4	X	X					
5	0101	1000	1011	1011	1000	1000001		5	X						X
6	0110	1001	1010	1100	1010	1000010		6	X					X	
7	0111	1010	1001	1101	1100	1000100		7	X			X			
8	1000	1011	1000	1110	1110	1001000		8	X		X				
9	1001	1111	1111	1111	1111	1010000		9	X		X				

Reflective Code

A code is said to be reflective when code for 9 is complement for the code for 0, and

so is for 8 and 1 codes, 7 and 2, 6 and 3, 5 and 4. Codes 2421, 5211, and excess-3 are reflective, whereas the 8421 code is not.

Sequential Codes

A code is said to be sequential when two subsequent codes, seen as numbers in binary representation, differ by one. This greatly aids mathematical manipulation of data. The 8421 and Excess-3 codes are sequential, whereas the 2421 and 5211 codes are not.

Non weighted codes

Non weighted codes are codes that are not positionally weighted. That is, each position within the binary number is not assigned a fixed value. Ex: Excess-3 code

Excess-3 Code

Excess-3 is a non weighted code used to express decimal numbers. The code derives its name from the fact that each binary code is the corresponding 8421 code plus 0011(3).

Gray Code

The gray code belongs to a class of codes called minimum change codes, in which only one bit in the code changes when moving from one code to the next. The Gray code is non-weighted code, as the position of bit does not contain any weight. The gray code is a reflective digital code which has the special property that any two subsequent numbers codes differ by only one bit. This is also called a unit- distance code. In digital Gray code has got a special place.

Decimal Number	Binary Code	Gray Code	Decimal Number	Binary Code	Gray Code
0	0000	0000	8	1000	1100
1	0001	0001	9	1001	1101
2	0010	0011	10	1010	1111
3	0011	0010	11	1011	1110
4	0100	0110	12	1100	1010
5	0101	0111	13	1101	1011
6	0110	0101	14	1110	1001
7	0111	0100	15	1111	1000

Binary to Gray Conversion

- Gray Code MSB is binary code MSB.
- Gray Code MSB-1 is the XOR of binary code MSB and MSB-1.
- MSB-2 bit of gray code is XOR of MSB-1 and MSB-2 bit of binary code.
- MSB-N bit of gray code is XOR of MSB-N-1 and MSB-N bit of binary code.

8421 BCD code (Natural BCD code):

Each decimal digit 0 through 9 is coded by a 4 bit binary no. called natural binary codes. Because of the 8,4,2,1 weights attached to it. It is a weighted code & also sequential . it is useful for mathematical operations. The advantage of this code is its ease of conversion to & from decimal. It is less efficient than the pure binary, it require more bits.

Ex: 14→1110 in binary

But as 0001 0100 in 8421 code.

The disadvantage of the BCD code is that , arithmetic operations are more complex than they are in pure binary . There are 6 illegal combinations 1010,1011,1100,1101,1110,1111 in these codes, they are not part of the 8421 BCD code system . The disadvantage of 8421 code is, the rules of binary addition 8421 no, but only to the individual 4 bit groups.

BCD Addition:

It is individually adding the corresponding digits of the decimal no,s expressed in 4 bit binary groups starting from the LSD . If there is no carry & the sum term is not an illegal code , no correction is needed .If there is a carry out of one group to the next group or if the sum term is an illegal code then $6_{10}(0100)$ is added to the sum term of that group & the resulting carry is added to the next group.

Ex: Perform decimal additions in 8421 code

(a)25+13

In BCD 25= 0010 0101

In BCD +13 =+0001 0011

38 0011 1000

No carry , no illegal code .This is the corrected sum

(b). 679.6 + 536.8

679.6 = 0110 0111 1001 .0110 in BCD
 +536.8 = +0101 0011 0010 .1000 in BCD

 1216.4 1011 1010 0110 . 1110 illegal codes
 +0110 + 0011 +0110 . + 0110 add 0110 to each

(1)0001 (1)0000 (1)0101 . (1)0100 propagate carry
 / / / /
 +1 +1 +1 +1

 0001 0010 0001 0110 . 0100

 1 2 1 6 . 4

BCD Subtraction:

Performed by subtracting the digits of each 4 bit group of the subtrahend the digits from the corresponding 4- bit group of the minuend in binary starting from the LSD . if there is no borrow from the next group , then $6_{10}(0110)$ is subtracted from the difference term of this group.

(a)38-15

In BCD 38= 0011 1000
 In BCD -15 = -0001 0101

 23 0010 0011

No borrow, so correct difference.

(b) 206.7-147.8

206.7 = 0010 0000 0110 . 0111 in BCD
 -147.8 = -0001 0100 0111 . 0110 in BCD

 58.9 0000 1011 1110 . 1111 borrows are present
 -0110 -0110 . -0110 subtract 0110

 0101 1000 . 1001

BCD Subtraction using 9's & 10's compliment methods:

Form the 9's & 10's compliment of the decimal subtrahend & encode that no. in the 8421 code . the resulting BCD no.s are then added.

EX: 305.5 – 168.8

	305.5 =	305.5			
	-168.8 =	+83.1		9's comp of -168.8	

		(1)136.6			
		+1		end around carry	
		136.7		corrected difference	
305.5 ₁₀ =	0011	0000 0101 .	0101		
	+831.1 ₁₀ =	+1000 0011 0001 .	0001	9's comp of 168. in BCD	

		+1011 0011 0110 .	0110	1011 is illegal code	
		+0110		add 0110	

		(1)0001 0011 0110 .	0110	+1 End around carry	

		0001 0011 0110 .	0111		
			= 136.7		

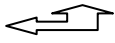
Excess three(xs-3)code:

It is a non-weighted BCD code .Each binary codeword is the corresponding 8421 codeword plus 0011(3).It is a sequential code & therefore , can be used for arithmetic operations..It is a self-complementing code.s o the subtraction by the method of compliment addition is more direct in xs-3 code than that in 8421 code. The xs-3 code has six invalid states 0000,0010,1101,1110,1111.. It has interesting properties when used in addition & subtraction.

Excess-3 Addition:

Add the xs-3 no.s by adding the 4 bit groups in each column starting from the LSD. If there is no carry starting from the addition of any of the 4-bit groups , subtract 0011 from the sum term of those groups (because when 2 decimal digits are added in xs-3 & there is no carry , result in xs-6). If there is a carry out, add 0011 to the sum term of those groups(because when there is a carry, the invalid states are skipped and the result is normal binary).

EX:	37	0110	1010	
	+28	+0101	1011	

	65	1011	(1)0101	carry generated
		+1		propagate carry

		1100	0101	add 0011 to correct 0101 &
		-0011	+0011	subtract 0011 to correct 1100

		1001	1000	=65 ₁₀

Excess -3 (XS-3) Subtraction:

Subtract the xs-3 no.s by subtracting each 4 bit group of the subtrahend from the corresponding 4 bit group of the minuend starting from the LSD .if there is no borrow from the next 4-bit group add 0011 to the difference term of such groups (because when decimal digits are subtracted in xs-3 & there is no borrow , result is normal binary). If there is a borrow , subtract 0011 from the differenceterm(b coz taking a borrow is equivalent to adding six invalid states , result is in xs-6)

Ex: 267-175

267 =	0101	1001	1010	
-175 =	-0100	1010	1000	

	0000	1111	0010	
	+0011	-0011	+0011	

	0011	1100	+0011	=92 ₁₀

Xs-3 subtraction using 9's & 10's compliment methods:

Subtraction is performed by the 9's compliment or 10's compliment

Ex:687-348 The subtrahend (348) xs -3 code & its compliment are:

9's comp of 348 = 651

Xs-3 code of 348 = 0110 0111 1011

1's comp of 348 in xs-3 = 1001 1000 0100

Xs=3 code of 348 in xs=3 = 1001 1000 0100

687 687
-348 → +651 9's compl of 348

339 (1)338
 +1 end around carry

 -
 339 corrected difference in decimal

1001 1011 1010 687 in xs-3
+1001 1000 0100 1's comp 348 in xs-3

-(1)0010(1)0011 1110 carry generated

//

+1 +1 propagate carry

(1)0011 0010 1110

+1 end around carry

0011 0011 1111 (correct 1111 by sub0011 and
+0011 +0011 +0011 correct both groups of 0011 by
----- ----- - - - adding 0011)

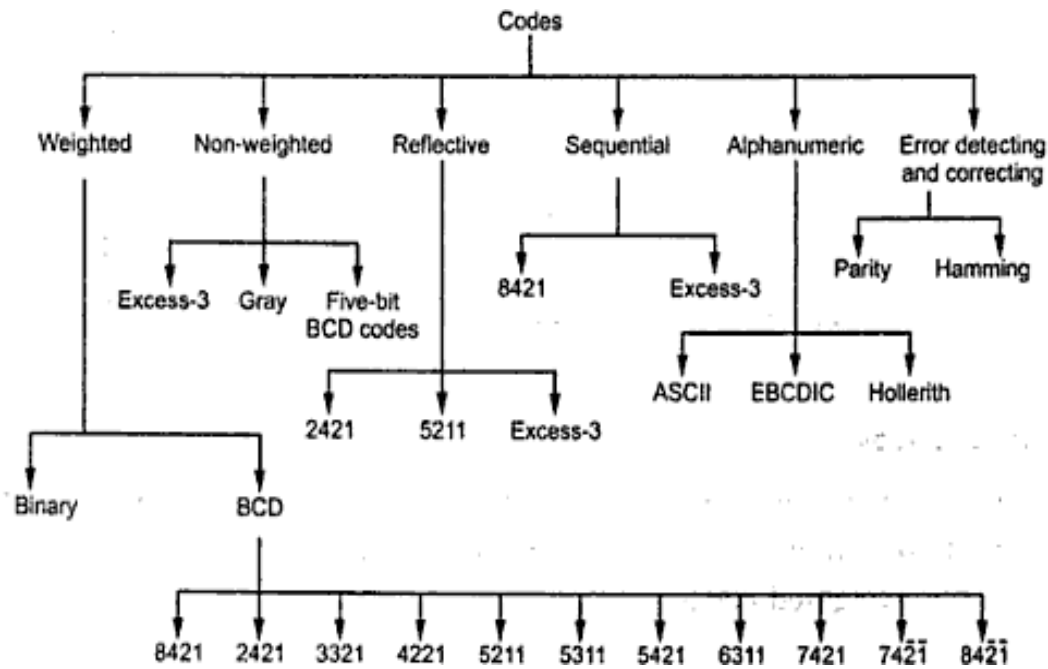
 -
0110 0110 1100 corrected diff in xs-3 = 330₁₀

The Gray code (reflective –code):

Gray code is a non-weighted code & is not suitable for arithmetic operations. It is not a BCD code . It is a cyclic code because successive code words in this code differ in one bit position only i.e, it is a unit distance code.Popular of the unit distance code.It is also a reflective code i.e,both reflective & unit distance. The n least significant bits for 2^n through $2^{n+1}-1$ are the mirror images of those for 0 through 2^n-1 .An N bit gray code can be obtained by reflecting an N-1 bit code about an axis at the end of the code, & putting the MSB of 0 above the axis & the MSB of 1 below the axis.

Reflection of graycodes:

Gray Code				Decimal	4 bit binary
1 bit	2 bit	3 bit	4 bit		
0	00	000	0000	0	0000
1	01	001	0001	1	0001
	11	011	0011	2	0010
	10	010	0010	3	0011
		110	0110	4	0100
		111	0111	5	0101
		101	0101	6	0110
		110	0100	7	0111
			1100	8	1000
			1101	9	1001
			1111	10	1010
			1110	11	1011
			1010	12	1100
			1011	13	1101
			1001	14	1110
			1000	15	1111



Binary codes block diagram

Error – Detecting codes: When binary data is transmitted & processed, it is susceptible to noise that can alter or distort its contents. The 1's may get changed to 0's & 1's. Because digital systems must be accurate to the digit, error can pose a problem. Several schemes have been devised to detect the occurrence of a single bit error in a binary word, so that whenever such an error occurs the concerned binary word can be corrected & retransmitted.

Parity: The simplest techniques for detecting errors is that of adding an extra bit known as parity bit to each word being transmitted. Two types of parity: Odd parity, even parity. For odd parity, the parity bit is set to a 0 or a 1 at the transmitter such that the total no. of 1 bit in the word including the parity bit is an odd no. For even parity, the parity bit is set to a 0 or a 1 at the transmitter such that the parity bit is an even no.

Decimal	8421 code	Odd parity	Even parity
0	0000	1	0
1	0001	0	1
2	0010	0	1
3	0011	1	0
4	0100	0	1
5	0100	1	0
6	0110	1	0
7	0111	0	1
8	1000	0	1
9	1001	1	0

When the digit data is received, a parity checking circuit generates an error signal if the total no of 1's is even in an odd parity system or odd in an even parity system. This parity check can always detect a single bit error but cannot detect 2 or more errors within the same word. Odd parity is used more often than even parity does not detect the situation. Where all 0's are created by a short ckt or some other fault condition.

Ex: Even parity scheme

(a) 10101010 (b) 11110110 (c) 10111001

Ans:

- (a) No. of 1's in the word is even is 4 so there is no error
- (b) No. of 1's in the word is even is 6 so there is no error
- (c) No. of 1's in the word is odd is 5 so there is error

Ex: odd parity









(a) 10110111 (b) 10011010 (c) 11101010

Ans:

- (a) No. of 1's in the word is even is 6 so word has error
- (b) No. of 1's in the word is even is 4 so word has error
- (c) No. of 1's in the word is odd is 5 so there is no error

Digital Logic Gates

Boolean functions are expressed in terms of AND, OR, and NOT operations, it is easier to implement a Boolean function with these type of gates.

Name	Graphic symbol	Algebraic function	Truth table															
AND		$F = x \cdot y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	x	y	F	0	0	0	0	1	0	1	0	0	1	1	1
x	y	F																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$F = x + y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	1
x	y	F																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
Inverter		$F = x'$	<table border="1"> <thead> <tr> <th>x</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td></tr> </tbody> </table>	x	F	0	1	1	0									
x	F																	
0	1																	
1	0																	
Buffer		$F = x$	<table border="1"> <thead> <tr> <th>x</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td></tr> </tbody> </table>	x	F	0	0	1	1									
x	F																	
0	0																	
1	1																	
NAND		$F = (xy)'$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	x	y	F	0	0	1	0	1	1	1	0	1	1	1	0
x	y	F																
0	0	1																
0	1	1																
1	0	1																
1	1	0																
NOR		$F = (x + y)'$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	0
x	y	F																
0	0	1																
0	1	0																
1	0	0																
1	1	0																
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </tbody> </table>	x	y	F	0	0	0	0	1	1	1	0	1	1	1	0
x	y	F																
0	0	0																
0	1	1																
1	0	1																
1	1	0																
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	<table border="1"> <thead> <tr> <th>x</th> <th>y</th> <th>F</th> </tr> </thead> <tbody> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </tbody> </table>	x	y	F	0	0	1	0	1	0	1	0	0	1	1	1
x	y	F																
0	0	1																
0	1	0																
1	0	0																
1	1	1																

Properties of XOR Gates

- XOR (also \oplus): the "not-equal" function
- $XOR(X, Y) = X \oplus Y = X'Y + XY'$
- Identities:
 - $X \oplus 0 = X$
 - $X \oplus 1 = X'$
 - $X \oplus X = 0$
 - $X \oplus X' = 1$
- Properties:
 - $X \oplus Y = Y \oplus X$
 - $(X \oplus Y) \oplus W = X \oplus (Y \oplus W)$

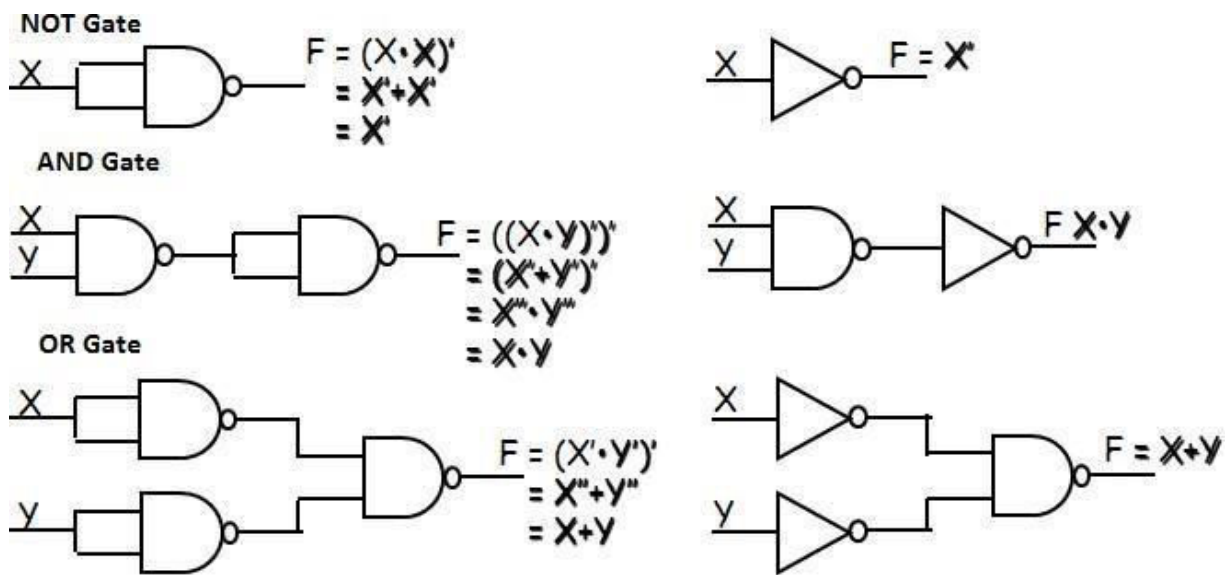
Universal Logic Gates

NAND and NOR gates are called Universal gates. All fundamental gates (NOT, AND, OR) can be realized by using either only NAND or only NOR gate. A universal gate provides flexibility and offers enormous advantage to logic designers.

NAND as a Universal Gate

NAND Known as a “universal” gate because ANY digital circuit can be implemented with NAND gates alone.

To prove the above, it suffices to show that AND, OR, and NOT can be implemented using NAND gates only.



Boolean Algebra: In 1854, George Boole developed an algebraic system now called Boolean algebra. In 1938, Claude E. Shannon introduced a two-valued Boolean algebra called switching algebra that represented the properties of bistable electrical switching circuits. For the formal definition of Boolean algebra, we shall employ the postulates formulated by E. V. Huntington in 1904.

Boolean algebra is a system of mathematical logic. It is an algebraic system consisting of the set of elements (0, 1), two binary operators called OR, AND, and one unary operator NOT. It is the basic mathematical tool in the analysis and synthesis of switching circuits. It is a way to express logic functions algebraically.

Boolean algebra, like any other deductive mathematical system, may be defined with a set of elements, a set of operators, and a number of unproved axioms or postulates. A *set* of elements is any collection of objects having a common property. If S is a set and x and y are certain objects, then $x \in S$ denotes that x is a member of the set S , and $y \notin S$ denotes that y is not an element of S . A set with a denumerable number of elements is specified by braces: $A = \{1, 2, 3, 4\}$, *i.e.* the elements of set A are the numbers 1, 2, 3, and 4. A *binary operator* defined on a set S of elements is a rule that assigns to each pair of elements from S a unique element from S . Example: In $a * b = c$, we say that $*$ is a binary operator if it specifies a rule for finding c from the pair (a, b) and also if $a, b, c \in S$.

Axioms and laws of Boolean algebra

Axioms or Postulates of Boolean algebra are a set of logical expressions that we accept without proof and upon which we can build a set of useful theorems.

	AND Operation	OR Operation	NOT Operation
Axiom1 :	$0 \cdot 0 = 0$	$0 + 0 = 0$	$0 = 1$
Axiom2:	$0 \cdot 1 = 0$	$0 + 1 = 1$	$1 = 0$
Axiom3:	$1 \cdot 0 = 0$	$1 + 0 = 1$	
Axiom4:	$1 \cdot 1 = 1$	$1 + 1 = 1$	

AND Law

Law1: $A \cdot 0 = 0$ (Null law)
 Law2: $A \cdot 1 = A$ (Identity law)
 Law3: $A \cdot A = A$ (Idempotence law)

OR Law

Law1: $A + 0 = A$
 Law2: $A + 1 = 1$
 Law3: $A + A = A$ (Idempotence law)

CLOSURE: The Boolean system is *closed* with respect to a binary operator if for every pair of Boolean values, it produces a Boolean result. For example, logical AND is closed in the Boolean system because it accepts only Boolean operands and produces only Boolean results.

_ A set S is closed with respect to a binary operator if, for every pair of elements of S , the binary operator specifies a rule for obtaining a unique element of S .

_ For example, the set of natural numbers $N = \{1, 2, 3, 4, \dots, 9\}$ is closed with respect to the binary operator plus (+) by the rule of arithmetic addition, since for any $a, b \in N$ we obtain a unique $c \in N$ by the operation $a + b = c$.

ASSOCIATIVE LAW:

A binary operator $*$ on a set S is said to be associative whenever $(x * y) * z = x * (y * z)$ for all $x, y, z \in S$, for all Boolean values x, y and z .

COMMUTATIVE LAW:

A binary operator $*$ on a set S is said to be commutative whenever $x * y = y * x$ for all $x, y, z \in S$

IDENTITY ELEMENT:

A set S is said to have an identity element with respect to a binary operation $*$ on S if there exists an element $e \in S$ with the property $e * x = x * e = x$ for every $x \in S$

BASIC IDENTITIES OF BOOLEAN ALGEBRA

- *Postulate 1(Definition):* A Boolean algebra is a closed algebraic system containing a set K of two or more elements and the two operators \cdot and $+$ which refer to logical AND and logical OR $x + 0 = x$
- $x \cdot 0 = 0$
- $x + 1 = 1$
- $x \cdot 1 = x$
- $x + x = x$
- $x \cdot x = x$
- $x + x' = 1$
- $x \cdot x' = 0$
- $x + y = y + x$
- $xy = yx$
- $x + (y + z) = (x + y) + z$
- $x(yz) = (xy)z$
- $x(y + z) = xy + xz$
- $x + yz = (x + y)(x + z)$
- $(x + y)' = x'y'$
- $(xy)' = x' + y'$

- $(x')' = x$

DeMorgan's Theorem

(a) $(a + b)' = a'b'$

(b) $(ab)' = a' + b'$

Generalized DeMorgan's Theorem

(a) $(a + b + \dots + z)' = a'b' \dots z'$

(b) $(a.b \dots z)' = a' + b' + \dots + z'$

Basic Theorems and Properties of Boolean algebra Commutative law

Law1: $A+B=B+A$

Law2: $A.B=B.A$

Associative law

Law1: $A + (B + C) = (A + B) + C$

Law2: $A(B.C) = (A.B)C$

Distributive law

Law1: $A.(B + C) = AB + AC$

Law2: $A + BC = (A + B).(A + C)$

Absorption law

Law1: $A + AB = A$

Law2: $A(A + B) = A$

Solution: $\frac{A(1+B)}{A}$

Solution: $\frac{A.A+A.B}{A+A.B}$
 $\frac{A(1+B)}{A}$

Consensus Theorem

Theorem1. $AB + A'C + BC = AB + A'C$ Theorem2. $(A+B).(A'+C).(B+C) = (A+B).(A'+C)$

The BC term is called the consensus term and is redundant. The consensus term is formed from a PAIR OF TERMS in which a variable (A) and its complement (A') are present; the consensus term is formed by multiplying the two terms and leaving out the selected variable and its complement

Consensus Theorem1 Proof:

$$AB + A'C + BC = AB + A'C + (A + A')BC$$

$$= AB + A'C + ABC + A'BC$$

$$=AB(1+C)+A'C(1+B)$$

$$= AB+ A'C$$

Principle of Duality

Each postulate consists of two expressions statement one expression is transformed into the other by interchanging the operations (+) and (·) as well as the identity elements 0 and 1. Such expressions are known as duals of each other.

If some equivalence is proved, then its dual is also immediately true.

If we prove: $(x.x)+(x'+x')=1$, then we have by duality: $(x+x)·(x'·x')=0$

The Huntington postulates were listed in pairs and designated by part (a) and part (b) in below table.

Table for Postulates and Theorems of Boolean algebra

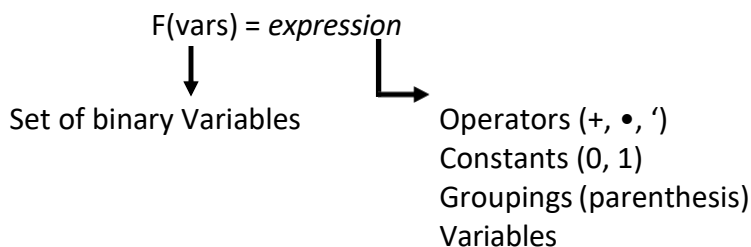
Part-A	Part-B
$A+0=A$	$A·0=0$
$A+1=1$	$A·1=A$
$A+A=A$ (Impotence law)	$A·A=A$ (Impotence law)
$A+\bar{A}=1$	$A·\bar{A}=0$
$\bar{\bar{A}}=A$ (double inversion law)	--
Commutative law: $A+B=B+A$	$A·B=B·A$
Associative law: $A+(B+C)=(A+B)+C$	$A(B·C)=(A·B)C$
Distributive law: $A·(B+C)=AB+AC$	$A+BC=(A+B)·(A+C)$
Absorption law: $A+AB=A$	$A(A+B)=A$
DeMorgan Theorem: $\overline{(A+B)} = \bar{A}·\bar{B}$	$\overline{(A·B)} = \bar{A}+\bar{B}$
Redundant Literal Rule: $A+\bar{A}B=A+B$	$A·(\bar{A}+B)=AB$
Consensus Theorem: $AB+A'C+BC=AB+A'C$	$(A+B)·(A'+C)·(B+C)=(A+B)·(A'+C)$

Boolean Function

Boolean algebra is an algebra that deals with binary variables and logic operations.

A Boolean function described by an algebraic expression consists of binary variables, the constants 0 and 1, and the logic operation symbols.

For a given value of the binary variables, the function can be equal to either 1 or 0.



Consider an example for the Boolean function

$$F1 = x + y'z$$

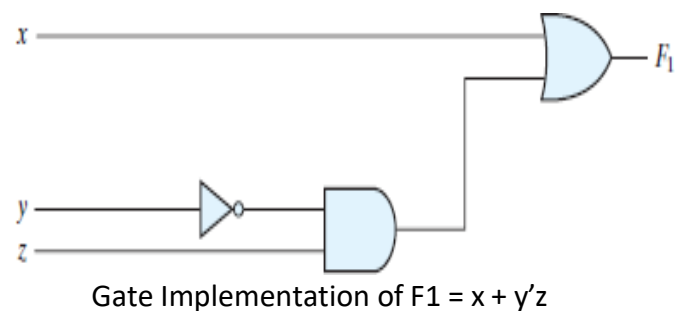
The function F_1 is equal to 1 if x is equal to 1 or if both y' and z are equal to 1. F_1 is equal to 0 otherwise. The complement operation dictates that when $y' = 1$, $y = 0$. Therefore, $F_1 = 1$ if $x = 1$ or if $y = 0$ and $z = 1$.

A Boolean function expresses the logical relationship between binary variables and is evaluated by determining the binary value of the expression for all possible values of the variables.

A Boolean function can be represented in a truth table. The number of rows in the truth table is 2^n , where n is the number of variables in the function. The binary combinations for the truth table are obtained from the binary numbers by counting from 0 through $2^n - 1$.

Truth Table for F_1

x	y	z	F_1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1



Note:

Q: Let a function $F()$ depend on n variables. How many rows are there in the truth table of $F()$?

A: 2^n rows, since there are 2^n possible binary patterns/combinations for the n variables.

Truth Tables

- Enumerates all possible combinations of variable values and the corresponding function value
- Truth tables for some arbitrary functions
 $F_1(x,y,z)$, $F_2(x,y,z)$, and $F_3(x,y,z)$ are shown to the below.

x	y	z	F_1	F_2	F_3
0	0	0	0	1	1
0	0	1	0	0	1

0	1	0	0	0	1
0	1	1	0	1	1
1	0	0	0	1	0
1	0	1	0	1	0
1	1	0	0	0	0
1	1	1	1	0	1

- Truth table: a unique representation of a Boolean function
- If two functions have identical truth tables, the functions are equivalent (and vice-versa).
- Truth tables can be used to prove equality theorems.
- However, the size of a truth table grows exponentially with the number of variables involved, hence unwieldy. This motivates the use of Boolean Algebra.

Boolean expressions-NOT unique

Unlike truth tables, expressions representing a Boolean function are NOT unique.

- Example:
 - $F(x,y,z) = x' \cdot y' \cdot z' + x' \cdot y \cdot z' + x \cdot y \cdot z'$
 - $G(x,y,z) = x' \cdot y' \cdot z' + y \cdot z'$
- The corresponding truth tables for F() and G() are to the right. They are identical.
- Thus, $F() = G()$

x	y	z	F	G
0	0	0	1	1
0	0	1	0	0
0	1	0	1	1
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	1
1	1	1	0	0

Algebraic Manipulation (Minimization of Boolean function)

- Boolean algebra is a useful tool for simplifying digital circuits.
- Why do it? Simpler can mean cheaper, smaller, faster.
- Example: Simplify $F = x'yz + x'yz' + xz$.

$$\begin{aligned}
 F &= x'yz + x'yz' + xz \\
 &= x'y(z+z') + xz \\
 &= x'y \cdot 1 + xz
 \end{aligned}$$

$$= x'y + xz$$

- Example: Prove

$$x'y'z' + x'yz' + xyz' = x'z' + yz'$$

- **Proof:**

$$\begin{aligned} x'y'z' + x'yz' + xyz' \\ &= x'y'z' + x'yz' + x'yz' + xyz' \\ &= x'z'(y'+y) + yz'(x'+x) \\ &= x'z' \cdot 1 + yz' \cdot 1 \\ &= x'z' + yz' \end{aligned}$$

Complement of a Function

- The complement of a function is derived by interchanging (\bullet and $+$), and (1 and 0), and complementing each variable.
- Otherwise, interchange 1s to 0s in the truth table column showing F.
- The *complement* of a function IS NOT THE SAME as the *dual* of a function.

Example

- Find $G(x,y,z)$, the complement of $F(x,y,z) = xy'z' + x'yz'$

$$\begin{aligned} \text{Ans: } G &= F' = (xy'z' + x'yz')' \\ &= (xy'z')' \bullet (x'yz')' \quad \text{DeMorgan} \\ &= (x'+y+z) \bullet (x+y'+z') \quad \text{DeMorgan again} \end{aligned}$$

Note: The complement of a function can also be derived by finding the function's *dual*, and then complementing all of the literals

Canonical and Standard Forms

We need to consider formal techniques for the simplification of Boolean functions.

Identical functions will have exactly the same canonical form.

- Minterms and Maxterms
- Sum-of-Minterms and Product-of- Maxterms
- Product and Sum terms
- Sum-of-Products (SOP) and Product-of-Sums (POS)

Definitions

Literal: A variable or its complement

Product term: literals connected by \bullet

Sum term: literals connected by $+$

Minterm: a product term in which all the variables appear exactly once, either complemented or uncomplemented.

Maxterm: a sum term in which all the variables appear exactly once, either complemented or uncomplemented.

Canonical form: Boolean functions expressed as a sum of Minterms or product of Maxterms are said to be in canonical form.

Minterm

- Represents exactly one combination in the truth table.
- Denoted by m_j , where j is the decimal equivalent of the minterm's corresponding binary combination (b_j).
- A variable in m_j is complemented if its value in b_j is 0, otherwise is uncomplemented.

Example: Assume 3 variables (A, B, C), and $j=3$. Then, $b_j = 011$ and its corresponding minterm is denoted by $m_j = A'BC$

Maxterm

- Represents exactly one combination in the truth table.
- Denoted by M_j , where j is the decimal equivalent of the maxterm's corresponding binary combination (b_j).
- A variable in M_j is complemented if its value in b_j is 1, otherwise is uncomplemented.

Example: Assume 3 variables (A, B, C), and $j=3$. Then, $b_j = 011$ and its corresponding maxterm is denoted by $M_j = A+B'+C'$

Truth Table notation for Minterms and Maxterms

- Minterms and Maxterms are easy to denote using a truth table.

Example: Assume 3 variables x,y,z (order is fixed)

x	y	z	Minterm	Maxterm
0	0	0	$x'y'z' = m_0$	$x+y+z = M_0$
0	0	1	$x'y'z = m_1$	$x+y+z' = M_1$
0	1	0	$x'yz' = m_2$	$x+y'+z = M_2$
0	1	1	$x'yz = m_3$	$x+y'+z' = M_3$
1	0	0	$xy'z' = m_4$	$x'+y+z = M_4$
1	0	1	$xy'z = m_5$	$x'+y+z' = M_5$
1	1	0	$xyz' = m_6$	$x'+y'+z = M_6$
1	1	1	$xyz = m_7$	$x'+y'+z' = M_7$

Canonical Forms

- Every function $F()$ has two canonical forms:
 - Canonical Sum-Of-Products (sum of minterms)
 - Canonical Product-Of-Sums (product of maxterms)

Canonical Sum-Of-Products:

The minterms included are those m_j such that $F() = 1$ in row j of the truth table for $F()$.

Canonical Product-Of-Sums:

The maxterms included are those M_j such that $F() = 0$ in row j of the truth table for $F()$.

Example

Consider a Truth table for $f_1(a,b,c)$ at right

The canonical sum-of-products form for f_1 is

$$f_1(a,b,c) = m_1 + m_2 + m_4 + m_6$$

$$= a'b'c + a'bc' + ab'c' + abc'$$

The canonical product-of-sums form for f_1 is

$$f_1(a,b,c) = M_0 \cdot M_3 \cdot M_5 \cdot M_7$$

$$= (a+b+c) \cdot (a+b'+c') \cdot (a'+b+c') \cdot (a'+b'+c').$$

- Observe that: $m_j = M_j'$

a	b	c	f_1
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Shorthand: Σ and Π

- $f_1(a,b,c) = \Sigma m(1,2,4,6)$, where Σ indicates that this is a sum-of-products form, and $m(1,2,4,6)$ indicates that the minterms to be included are m_1 , m_2 , m_4 , and m_6 .
- $f_1(a,b,c) = \Pi M(0,3,5,7)$, where Π indicates that this is a product-of-sums form, and $M(0,3,5,7)$ indicates that the maxterms to be included are M_0 , M_3 , M_5 , and M_7 .
- Since $m_j = M_j'$ for any j ,
 $\Sigma m(1,2,4,6) = \Pi M(0,3,5,7) = f_1(a,b,c)$
-

Conversion between Canonical Forms

- Replace Σ with Π (or *vice versa*) and replace those j 's that appeared in the original form with those that do not.
 - Example:

$$\begin{aligned} f_1(a,b,c) &= a'b'c + a'bc' + ab'c' + abc' \\ &= m_1 + m_2 + m_4 + m_6 \\ &= \Sigma(1,2,4,6) \\ &= \Pi(0,3,5,7) \\ &= (a+b+c) \cdot (a+b'+c') \cdot (a'+b+c') \cdot (a'+b'+c') \end{aligned}$$

Standard Forms

Another way to express Boolean functions is in standard form. In this configuration, the terms that form the function may contain one, two, or any number of literals.

There are two types of standard forms: the sum of products and products of sums.

The sum of products is a Boolean expression containing AND terms, called product terms, with one or more literals each. The sum denotes the ORing of these terms. An example of a function expressed as a sum of products is

$$F1 = y' + xy + x'yz'$$

The expression has three product terms, with one, two, and three literals. Their sum is, in effect, an OR operation.

A product of sums is a Boolean expression containing OR terms, called sum terms. Each term may have any number of literals. The product denotes the ANDing of these terms. An example of a function expressed as a product of sums is

$$F2 = x(y' + z)(x' + y + z')$$

This expression has three sum terms, with one, two, and three literals. The product is an AND operation.

Conversion of SOP from standard to canonical form

Example-1.

Express the Boolean function $F = A + B'C$ as a sum of minterms.

Solution: The function has three variables: A, B, and C. The first term A is missing two variables; therefore,

$$A = A(B + B') = AB + AB'$$

This function is still missing one variable, so

$$\begin{aligned} A &= AB(C + C') + AB'(C + C') \\ &= ABC + ABC' + AB'C + AB'C' \end{aligned}$$

The second term $B'C$ is missing one variable; hence,

$$B'C = B'C(A + A') = AB'C + A'B'C$$

Combining all terms, we have

$$\begin{aligned} F &= A + B'C \\ &= ABC + ABC' + AB'C + AB'C' + A'B'C \end{aligned}$$

But $AB'C$ appears twice, and according to theorem $(x + x = x)$, it is possible to remove one of those occurrences. Rearranging the minterms in ascending order, we finally obtain

$$\begin{aligned} F &= A'B'C + AB'C + ABC' + ABC \\ &= m_1 + m_4 + m_5 + m_6 + m_7 \end{aligned}$$

When a Boolean function is in its sum-of-minterms form, it is sometimes convenient to express the function in the following brief notation:

$$F(A, B, C) = \sum m(1, 4, 5, 6, 7)$$

Example-2.

Express the Boolean function $F = xy + x'z$ as a product of maxterms.

Solution: First, convert the function into OR terms by using the distributive law:

$$\begin{aligned} F &= xy + x'z = (xy + x')(xy + z) \\ &= (x + x')(y + x')(x + z)(y + z) \\ &= (x' + y)(x + z)(y + z) \end{aligned}$$

The function has three variables: x, y, and z. Each OR term is missing one variable; therefore,

$$\begin{aligned} x' + y &= x' + y + zz' = (x' + y + z)(x' + y + z') \\ x + z &= x + z + yy' = (x + y + z)(x + y' + z) \\ y + z &= y + z + xx' = (x + y + z)(x' + y + z) \end{aligned}$$

Combining all the terms and removing those which appear more than once, we finally obtain

$$\begin{aligned} F &= (x + y + z)(x + y' + z)(x' + y + z)(x' + y + z) \\ F &= M_0 M_2 M_4 M_5 \end{aligned}$$

A convenient way to express this function is as

$$\text{follows: } F(x, y, z) = \pi M(0, 2, 4, 5)$$

The product symbol, π , denotes the ANDing of maxterms; the numbers are the indices of the maxterms of the function.

Two-variable k-map:

A two-variable k-map can have $2^2=4$ possible combinations of the input variables A and B. Each of these combinations, $\bar{A}\bar{B}$, $\bar{A}B$, $A\bar{B}$, AB (in the SOP form) is called a minterm. The minterm may be represented in terms of their decimal designations – m_0 for $\bar{A}\bar{B}$, m_1 for $\bar{A}B$, m_2 for $A\bar{B}$ and m_3 for AB , assuming that A represents the MSB. The letter m stands for minterm and the subscript represents the decimal designation of the minterm. The presence or absence of a minterm in the expression indicates that the output of the logic circuit assumes logic 1 or logic 0 level for that combination of input variables.

The expression $f = \bar{A}\bar{B} + A\bar{B} + AB$, it can be expressed using min

$$\text{term as } F = m_0 + m_2 + m_3 = \sum m(0, 2, 3)$$

Using Truth Table:

Minterm	Inputs		Output F
	A	B	
0	0	0	1
1	0	1	0
2	1	0	1
3	1	1	1

A 1 in the output contains that particular minterm in its sum and a 0 in that column indicates that the particular minterm does not appear in the expression for output. This information can also be indicated by a two-variable k-map.

Mapping of SOP Expressions:

A two-variable k-map has $2^2=4$ squares. These squares are called cells. Each square on the k-map represents a unique minterm. The minterm designation of the squares are placed in any square, indicates that the corresponding minterm does output expressions. And a 0 or no entry in any square indicates that the corresponding minterm does not appear in the expression for output.

	B	0	1
A	0	A'B'	A'B
	1	AB'	AB

The minterms of a two-variable k-map

The mapping of the expressions $=\sum m(0,2,3)$ is

	B	0	1
A		⁰	¹
0	1	0	
1	1	1	

k-map of $\sum m(0,2,3)$

EX: Map the expressions $f = B + A$

$F = m_1 + m_2 = \sum m(1,2)$ The k-map is

	B	0	1
A		⁰	¹
0	0	1	
1	1	0	

Minimizations of SOP expressions:

To minimize Boolean expressions given in the SOP form by using the k-map, look for adjacent adjacent squares having 1's minterms adjacent to each other, and combine them to form larger squares to eliminate some variables. Two squares are said to be adjacent to each other, if their minterms differ in only one variable. (i.e, B & A differ only in one variable. so they may be combined to form a 2-square to eliminate the variable B. similarly all other.

The necessary condition for adjacency of minterms is that their decimal designations must differ by a power of 2. A minterm can be combined with any number of minterms adjacent to it to form larger squares. Two minterms which are adjacent to each other can be combined to form a bigger square called a 2-square or a pair. This eliminates one variable – the variable that is not common to both the minterms. For EX:

m_0 and m_1 can be combined to yield,

$$f_1 = m_0 + m_1 = \bar{A}B + A\bar{B} = (B + \bar{A})$$

m_0 and m_2 can be combined to yield,

$$f_2 = m_0 + m_2 = \bar{A}\bar{B} + A\bar{B} = (\bar{B} + \bar{A})$$

m_1 and m_3 can be combined to yield,

$$f_3 = m_1 + m_3 = B + AB = B(+) = B$$

m_2 and m_3 can be combined to yield,

$$f_4 = m_2 + m_3 = A + AB = A(B +) = A$$

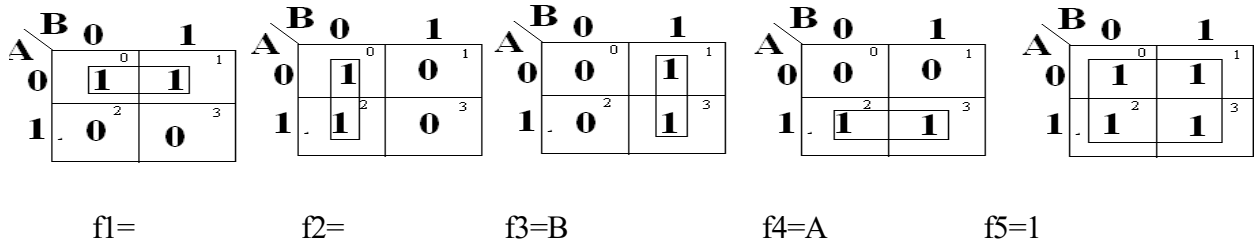
m_0, m_1, m_2 and m_3 can be combined to yield,

$$= + + A + AB$$

$$= (B +) + A(B +)$$

$$= + A$$

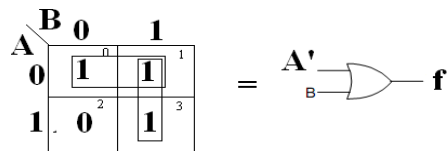
$$= 1$$



The possible minterm groupings in a two-variable k-map.

Two 2-squares adjacent to each other can be combined to form a 4-square. A 4-square eliminates 2 variables. A 4-square is called a quad. To read the squares on the map after minimization, consider only those variables which remain constant through the square, and ignore the variables which are varying. Write the non complemented variable if the variable is remaining constant as a 1, and the complemented variable if the variable is remaining constant as a 0, and write the variables as a product term. In the above figure f_1 read as A' , because, along the square, A remains constant as a 0, that is, as A' , whereas B is changing from 0 to 1.

EX: Reduce the minterm $f = A' + AB$ using mapping Expressed in terms of minterms, the given expression is $F = m_0 + m_1 + m_2 + m_3 = \sum(0,1,3)$ & the figure shows the k-map for f and its reduction. In one 2-square, A is constant as a 0 but B varies from a 0 to a 1, and in the other 2-square, B is constant as a 1 but A varies from a 0 to a 1. So, the reduced expressions is $A' + B$.



It requires two gate inputs for realization as

$$f = A' + B \quad (\text{k-map in SOP form, and logic diagram.})$$

The main criterion in the design of a digital circuit is that its cost should be as low as possible. For that the expression used to realize that circuit must be minimal. Since the cost is proportional to number of gate inputs in the circuit, an expression is considered minimal only if it corresponds to the least possible number of gate inputs. & there is no guarantee for that k-map in SOP is the real minimal. To obtain real minimal expression, obtain the minimal expression both in SOP & POS form by using k-maps and take the minimal of these two minimal.

The 1's on the k-map indicate the presence of minterms in the output expressions, where as the 0s indicate the absence of minterms. Since the absence of a minterm in the SOP expression means the presence of the corresponding maxterm in the POS expression of the same. when a SOP expression is plotted on the k-map, 0s or no entries on the k-map represent the maxterms. To obtain the minimal expression in the POS form, consider the 0s on the k-map and follow the procedure used for combining 1s. Also, since the absence of a maxterm in the POS expression means the presence of the corresponding minterm in the SOP expression of the same, when a POS expression is plotted on the k-map, 1s or no entries on the k-map represent the minterms.

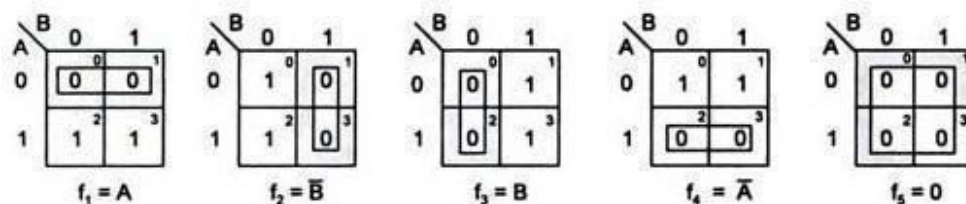
Mapping of POS expressions:

Each sum term in the standard POS expression is called a maxterm. A function in two variables (A, B) has four possible maxterms, $A+B, A+\bar{B}, \bar{A}+B, \bar{A}+\bar{B}$

. They are represented as $M_0, M_1, M_2,$ and M_3 respectively. The uppercase letter M stands for maxterm and its subscript denotes the decimal designation of that maxterm obtained by treating the non-complemented variable as a 0 and the complemented variable as a 1 and putting them side by side for reading the decimal equivalent of the binary number so formed.

For mapping a POS expression on to the k-map, 0s are placed in the squares corresponding to the maxterms which are presented in the expression and 1s are placed in the squares corresponding to the maxterm which are not present in the expression. The decimal designation of the squares of the squares for maxterms is the same as that for the minterms. A two-variable k-map & the associated maxterms are as the maxterms of a two-variable k-map

The possible maxterm groupings in a two-variable k-map



Minimization of POS Expressions:

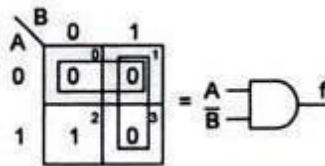
To obtain the minimal expression in POS form, map the given POS expression on to the K-map and combine the adjacent 0s into as large squares as possible. Read the squares putting the complemented variable if its value remains constant as a 1 and the non-complemented variable if its value remains constant as a 0 along the entire square (ignoring the variables which do not remain constant throughout the square) and then write them as a sum term.

Various maxterm combinations and the corresponding reduced expressions are shown in figure. In this f_1 read as A because A remains constant as a 0 throughout the square and B changes from a 0 to a 1. f_2 is read as B' because B remains constant along the square as a 1 and A changes from a 0 to a 1. f_3

Is read as a 0 because both the variables are changing along the square.

Ex: Reduce the expression $f=(A+B)(A+B')(A'+B')$ using mapping.

The given expression in terms of maxterms is $f=\pi M(0,1,3)$. It requires two gates inputs for realization of the reduced expression as



$$F=AB'$$

K-map in POS form and logic diagram

In this given expression ,the maxterm M_2 is absent. This is indicated by a 1 on the k-map. The corresponding SOP expression is $\sum m_2$ or AB' . This realization is the same as that for the POS form.

Three-variable K-map:

A function in three variables (A, B, C) expressed in the standard SOP form can have eight possible combinations: $ABC, AB\bar{C}, A\bar{B}C, \bar{A}BC, AB\bar{C}, \bar{A}B\bar{C}, \bar{A}\bar{B}C, \bar{A}\bar{B}\bar{C}$, and ABC . Each one of these combinations designate d by $m_0, m_1, m_2, m_3, m_4, m_5, m_6,$ and m_7 , respectively, is called a minterm. A is the MSB of the minterm designator and C is the LSB.

In the standard POS form, the eight possible combinations are: $A+B+C, A+B+\bar{C}, A+B+\bar{C}, A+B+\bar{C}, A+B+\bar{C}, A+B+\bar{C}, A+B+\bar{C}, A+B+\bar{C}$. Each one of these combinations designated by $M_0, M_1, M_2, M_3, M_4, M_5, M_6,$ and M_7 respectively is called a maxterm. A is the MSB of the maxterm designator and C is the LSB.

A three-variable k-map has, therefore, $8(=2^3)$ squares or cells, and each square on the map represents a minterm or maxterm as shown in figure. The small number on the top right corner of each cell indicates the minterm or maxterm designation.

	BC	00	01	11	10
A	0	$\bar{A}\bar{B}\bar{C}$ ⁰	$\bar{A}\bar{B}C$ ¹	$\bar{A}BC$ ³	$\bar{A}B\bar{C}$ ²
	1	$A\bar{B}\bar{C}$ ⁴	$A\bar{B}C$ ⁵	ABC ⁷	$AB\bar{C}$ ⁶

(a) Minterms

	BC	00	01	11	10
A	0	$A+B+C$ ⁰	$A+B+\bar{C}$ ¹	$A+\bar{B}+\bar{C}$ ³	$A+\bar{B}+C$ ²
	1	$\bar{A}+B+C$ ⁴	$\bar{A}+B+\bar{C}$ ⁵	$\bar{A}+\bar{B}+\bar{C}$ ⁷	$\bar{A}+\bar{B}+C$ ⁶

(b) Maxterms

The three-variable k-map.

The binary numbers along the top of the map indicate the condition of B and C for each column. The binary number along the left side of the map against each row indicates the condition of A for that row. For example, the binary number 01 on top of the second column in fig indicates that the variable B appears in complemented form and the variable C in non-complemented form in all the minterms in that column. The binary number 0 on the left of the first row indicates that the variable A appears in complemented form in all the minterms in that row, the binary numbers along the top of the k-map are not in normal binary order. They are, infact, in the Gray code. This is to ensure that twophysically adjacent squares are really adjacent, i.e., their minterms or maxterms differ by only one variable.

Ex: Map the expression $f = C + \dots + \dots + ABC$

In the given expression , the minterms are : $C=001=m_1$; $=101=m_5$;
 $=010=m_2$;

$=110=m_6$; $ABC=111=m_7$.

So the expression is $f = \sum m(1,5,2,6,7) = \sum m(1,2,5,6,7)$. The corresponding k-map is

	BC	00	01	11	10
A	0	0 ⁰	1 ¹	0 ³	1 ²
	1	0 ⁴	1 ⁵	1 ⁷	1 ⁶

K-map in SOP form

Ex: Map the expression $f = (A+B+C)(\dots)(\dots)(A+\dots)(\dots)$

In the given expression the maxterms are
 $:A+B+C=000=M_0$; $\dots = 101=M_5$; $\dots = 111=M_7$; $A+\dots = 011=M_3$; \dots
 $=110=M_6$.

So the expression is $f = \pi M(0,5,7,3,6) = \pi M(0,3,5,6,7)$. The mapping of the expression is

	BC	00	01	11	10
A					
0		0 ⁰	1 ¹	0 ³	1 ²
1		1 ⁴	0 ⁵	0 ⁷	0 ⁶

K-map in POS form.

Minimization of SOP and POS expressions:

For reducing the Boolean expressions in SOP (POS) form plotted on the k-map, look at the 1s (0s) present on the map. These represent the minterms (maxterms). Look for the minterms (maxterms) adjacent to each other, in order to combine them into larger squares. Combining of adjacent squares in a k-map containing 1s (or 0s) for the purpose of simplification of a SOP (or POS) expression is called *looping*. Some of the minterms (maxterms) may have many adjacencies. Always start with the minterms (maxterm) with the least number of adjacencies and try to form as large as large a square as possible. The larger must form a geometric square or rectangle. They can be formed even by wrapping around, but cannot be formed by using diagonal configurations. Next consider the minterm (maxterm) with next to the least number of adjacencies and form as large a square as possible. Continue this till all the minterms (maxterms) are taken care of . A minterm (maxterm) can be part of any number of squares if it is helpful in reduction. Read the minimal expression from the k-map, corresponding to the squares formed. There can be more than one minimal expression.

Two squares are said to be adjacent to each other (since the binary designations along the top of the map and those along the left side of the map are in Gray code), if they are physically adjacent to each other, or can be made adjacent to each other by wrapping around. For squares to be combinable into bigger squares it is essential but not sufficient that their minterm designations must differ by a power of two.

General procedure to simplify the Boolean expressions:

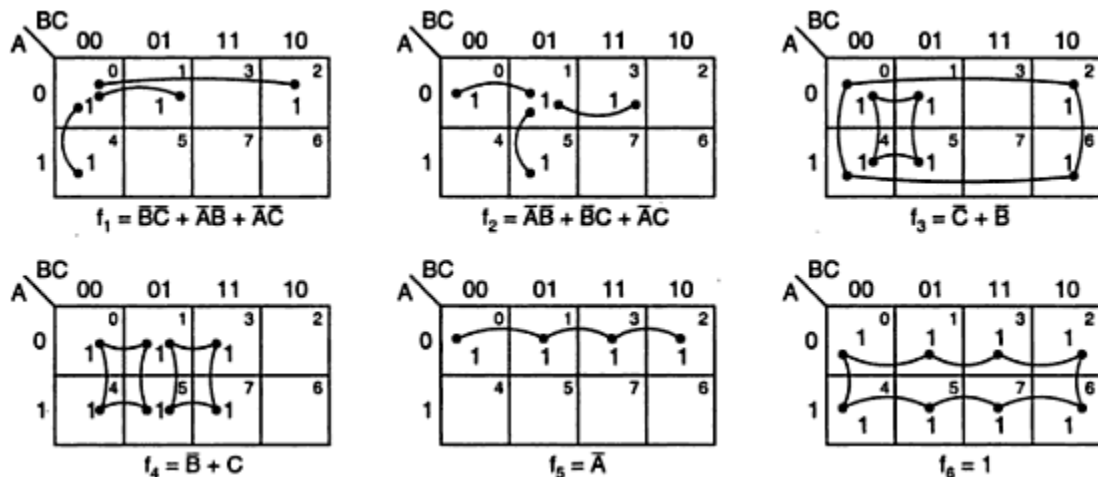
1. Plot the k-map and place 1s(0s) corresponding to the minterms (maxterms) of the SOP (POS) expression.
2. Check the k-map for 1s(0s) which are not adjacent to any other 1(0). They are isolated minterms(maxterms) . They are to be read as they are because they cannot be combined even into a 2-square.
3. Check for those 1s(0s) which are adjacent to only one other 1(0) and make them pairs (2 squares).
4. Check for quads (4 squares) and octets (8 squares) of adjacent 1s (0s) even if they contain some 1s(0s) which have already been combined. They must geometrically form a square or a rectangle.
5. Check for any 1s(0s) that have not been combined yet and combine them into bigger squares if possible.
6. Form the minimal expression by summing (multiplying) the product the product (sum) terms of all the groups.

Reading the K-maps:

While reading the reduced k-map in SOP (POS) form, the variable which remains constant as 0 along the square is written as the complemented (non-complemented) variable and the one which remains constant as 1 along the square is written as non-complemented (complemented) variable and the term as a product (sum) term. All the product (sum) terms are added (multiplied).

Some possible combinations of minterms and the corresponding minimal expressions read from the k-maps are shown in fig: Here f_6 is read as 1, because along the 8-square no variable remains constant. f_5 is read as \bar{A} , because, along the 4-square formed by m_0, m_1, m_4 and m_5 , the variables B and C are changing, and A remains constant as a 0. Algebraically,

$$\begin{aligned}
 f_5 &= m_0 + m_1 + m_4 + m_5 \\
 &= \bar{A} + \bar{A}C + \bar{A}B + \bar{A}BC \\
 &= \bar{A}(\bar{C} + C) + \bar{A}(B + BC) \\
 &= \bar{A} + \bar{A}B \\
 &= \bar{A}(1 + B) = \bar{A}
 \end{aligned}$$

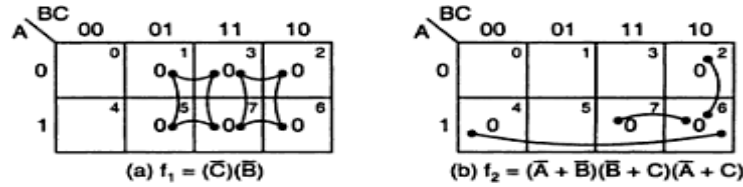


f_3 is read as $\bar{C} + \bar{B}$, because in the 4-square formed by m_0, m_2, m_6 , and m_4 , the variable A and B are changing, whereas the variable C remains constant as a 0. So it is read as \bar{C} . In the 4-square formed by m_0, m_1, m_4, m_5 , A and C are changing but B remains constant as a 0. So it is read as \bar{B} . So, the resultant expression for f_3 is the sum of these two, i.e., $\bar{C} + \bar{B}$.

f_1 is read as $\bar{B}\bar{C} + A\bar{B} + A\bar{C}$, because in the 2-square formed by m_0 and m_4 , A is changing from a 0 to a 1. Whereas B and C remain constant as a 0. So it is read as $\bar{B}\bar{C}$. In the 2-square formed by m_0 and m_1 , C is changing from a 0 to a 1, whereas A and B remain constant as a 0. So it is read as $A\bar{B}$. In the 2-square formed by m_0 and m_2 , B is changing from a 0 to a 1 whereas A and C remain constant as a 0. So, it is read as $A\bar{C}$. Therefore, the resultant SOP expression is

$$\bar{B}\bar{C} + A\bar{B} + A\bar{C}$$

Some possible maxterm groupings and the corresponding minimal POS expressions read from the k-map are



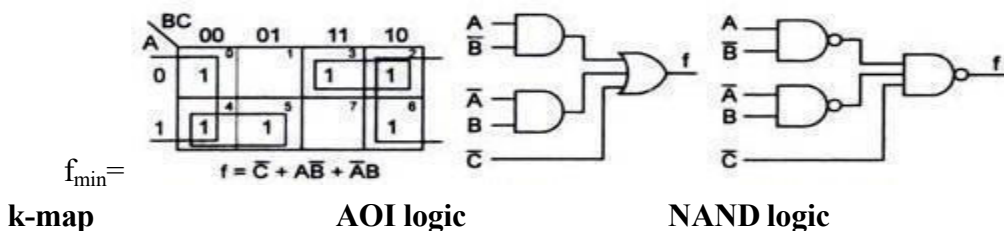
In this figure, along the 4-square formed by M_1, M_3, M_7, M_5 , A and B are changing from a 0 to a 1, where as C remains constant as a 1. SO it is read as \bar{C} . Along the 4-squad formed by M_3, M_2, M_7 , and M_6 , variables A and C are changing from a 0 to a 1. But B remains constant as a 1. So it is read as \bar{B} . The minimal expression is the product of these two terms, i.e., $f_1 = (\bar{C})(\bar{B})$.also in this figure, along the 2-square formed by M_4 and M_6 , variable B is changing from a 0 to a 1, while variable A remains constant as a 1 and variable C remains constant as a 0. SO, read it as $A\bar{C}$.

Similarly, the 2-square formed by M_7 and M_6 is read as $A\bar{C}$, while the 2-square formed by M_2 and M_6 is read as $A\bar{C}$. The minimal expression is the product of these sum terms, i.e., $f_2 = (A\bar{C}) + (A\bar{C}) + (A\bar{C})$

Ex:Reduce the expression $f = \sum m(0,2,3,4,5,6)$ using mapping and implement it in AOI logic as well as in NAND logic.The Sop k-map and its reduction, and the implementation of the minimal expression using AOI logic and the corresponding NAND logic are shown in figures below

In SOP k-map, the reduction is done as:

- m_5 has only one adjacency m_4 , so combine m_5 and m_4 into a square. Along this 2-square A remains constant as 1 and B remains constant as 0 but C varies from 0 to 1. So read it as $A\bar{B}$.
- m_3 has only one adjacency m_2 , so combine m_3 and m_2 into a square. Along this 2-square A remains constant as 0 and B remains constant as 1 but C varies from 1 to 0. So read it as $\bar{A}B$.
- m_6 can form a 2-square with m_2 and m_4 can form a 2-square with m_0 , but observe that by wrapping the map from left to right m_0, m_4, m_2, m_6 can form a 4-square. Out of these m_2 and m_4 have already been combined but they can be utilized again. So make it. Along this 4-square, A is changing from 0 to 1 and B is also changing from 0 to 1 but C is remaining constant as 0. so read it as \bar{C} .
- Write all the product terms in SOP form. So the minimal SOP expression is



Four variable k-maps:

Four variable k-map expressions can have $2^4=16$ possible combinations of input variables such as $\bar{A}\bar{B}\bar{C}\bar{D}$, $\bar{A}\bar{B}\bar{C}D$, $\bar{A}\bar{B}C\bar{D}$, $\bar{A}\bar{B}CD$ with minterm designations m_0, m_1, \dots, m_{15} respectively in SOP form & $A+B+C+D, A+B+C+\bar{D}, \dots, \bar{A}+\bar{B}+\bar{C}+\bar{D}$ with maxterms M_0, M_1, \dots, M_{15} respectively in POS form. It has $2^4=16$ squares or cells. The binary number designations of rows & columns are in the gray code. Here follows 01 & 10 follows 11 called Adjacency ordering.

		CD			
		00	01	11	10
AB	00	0 $\bar{A}\bar{B}\bar{C}\bar{D}$	1 $\bar{A}\bar{B}\bar{C}D$	3 $\bar{A}\bar{B}C\bar{D}$	2 $\bar{A}\bar{B}CD$
	01	4 $\bar{A}B\bar{C}\bar{D}$	5 $\bar{A}B\bar{C}D$	7 $\bar{A}BC\bar{D}$	6 $\bar{A}BCD$
	11	12 $A\bar{B}\bar{C}\bar{D}$	13 $A\bar{B}\bar{C}D$	15 $A\bar{B}C\bar{D}$	14 $A\bar{B}CD$
	10	8 $AB\bar{C}\bar{D}$	9 $AB\bar{C}D$	11 $ABC\bar{D}$	10 $ABCD$

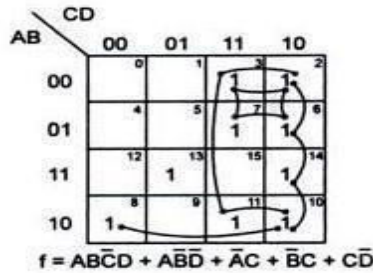
SOP form

		CD			
		00	01	11	10
AB	00	0 $A+B+C+D$	1 $A+B+C+\bar{D}$	3 $A+B+\bar{C}+\bar{D}$	2 $A+B+\bar{C}+D$
	01	4 $A+\bar{B}+C+D$	5 $A+\bar{B}+C+\bar{D}$	7 $A+\bar{B}+\bar{C}+\bar{D}$	6 $A+\bar{B}+\bar{C}+D$
	11	12 $\bar{A}+\bar{B}+C+D$	13 $\bar{A}+\bar{B}+C+\bar{D}$	15 $\bar{A}+\bar{B}+\bar{C}+\bar{D}$	14 $\bar{A}+\bar{B}+\bar{C}+D$
	10	8 $\bar{A}+B+C+D$	9 $\bar{A}+B+C+\bar{D}$	11 $\bar{A}+B+\bar{C}+\bar{D}$	10 $\bar{A}+B+\bar{C}+D$

POS form

EX: Reduce using mapping the expression $\Sigma m(2, 3, 6, 7, 8, 10, 11, 13, 14)$.

Start with the minterm with the least number of adjacencies. The minterm m_{13} has no adjacency. Keep it as it is. The m_8 has only one adjacency, m_{10} . Expand m_8 into a 2-square with m_{10} . The m_7 has two adjacencies, m_6 and m_3 . Hence m_7 can be expanded into a 4-square with m_6, m_3 and m_2 . Observe that, m_7, m_6, m_2 , and m_3 form a geometric square. The m_{11} has 2 adjacencies, m_{10} and m_3 . Observe that, m_{11}, m_{10}, m_3 , and m_2 form a geometric square on wrapping the K-map. So expand m_{11} into a 4-square with m_{10}, m_3 and m_2 . Note that, m_2 and m_3 have already become a part of the 4-square m_7, m_6, m_2 , and m_3 . But if m_{11} is expanded only into a 2-square with m_{10} , only one variable is eliminated. So m_2 and m_3 are used again to make another 4-square with m_{11} and m_{10} to eliminate two variables. Now only m_6 and m_{14} are left uncovered. They can form a 2-square that eliminates only one variable. Don't do that. See whether they can be expanded into a larger square. Observe that, m_2, m_6, m_{14} , and m_{10} form a rectangle. So m_6 and m_{14} can be expanded into a 4-square with m_2 and m_{10} . This eliminates two variables.



Five variable k-map:

Five variable k-map can have $2^5 = 32$ possible combinations of input variable as $00000, 00001, \dots, 11111$ with minterms m_0, m_1, \dots, m_{31} respectively in SOP & $A+B+C+D+E, A+B+C+\bar{D}+\bar{E}, \dots, \bar{A}+\bar{B}+\bar{C}+\bar{D}+\bar{E}$ with maxterms M_0, M_1, \dots, M_{31} respectively in POS form. It has $2^5 = 32$ squares or cells of the k-map are divided into 2 blocks of

16 squares each. The left block represents minterms from m_0 to m_{15} in which A is a 0, and the right block represents minterms from m_{16} to m_{31} in which A is 1. The 5-variable k-map may contain 2-squares, 4-squares, 8-squares, 16-squares or 32-squares involving these two blocks. Squares are also considered adjacent in these two blocks, if when superimposing one block on top of another, the squares coincide with one another.

Some possible 2-squares in a five-variable map are $m_0, m_{16}; m_2, m_{18}; m_4, m_{20}; m_6, m_{22}; m_8, m_{24}; m_{10}, m_{26}; m_{12}, m_{28}; m_{14}, m_{30}; m_{15}, m_{31}; m_{11}, m_{27}$.

Some possible 4-squares are $m_0, m_2, m_{16}, m_{18}; m_0, m_4, m_{16}, m_{20}; m_0, m_8, m_{16}, m_{24}; m_0, m_{12}, m_{16}, m_{28}; m_4, m_6, m_{20}, m_{22}; m_4, m_{12}, m_{20}, m_{28}; m_8, m_{10}, m_{24}, m_{26}; m_8, m_{16}, m_{24}, m_{32}; m_{12}, m_{14}, m_{28}, m_{30}; m_{12}, m_{20}, m_{28}, m_{32}; m_{16}, m_{18}, m_{32}, m_{34}; m_{16}, m_{24}, m_{32}, m_{36}; m_{20}, m_{22}, m_{36}, m_{38}; m_{20}, m_{28}, m_{36}, m_{40}; m_{24}, m_{26}, m_{40}, m_{42}; m_{24}, m_{32}, m_{40}, m_{44}; m_{28}, m_{30}, m_{44}, m_{46}; m_{28}, m_{36}, m_{44}, m_{50}; m_{32}, m_{34}, m_{50}, m_{52}; m_{32}, m_{40}, m_{52}, m_{56}; m_{36}, m_{38}, m_{56}, m_{60}; m_{36}, m_{44}, m_{60}, m_{64}; m_{40}, m_{42}, m_{64}, m_{68}; m_{40}, m_{48}, m_{68}, m_{72}; m_{44}, m_{46}, m_{72}, m_{76}; m_{44}, m_{52}, m_{76}, m_{80}; m_{48}, m_{50}, m_{80}, m_{84}; m_{48}, m_{56}, m_{84}, m_{88}; m_{52}, m_{54}, m_{88}, m_{92}; m_{52}, m_{60}, m_{92}, m_{96}; m_{56}, m_{58}, m_{96}, m_{100}; m_{56}, m_{64}, m_{100}, m_{104}; m_{60}, m_{62}, m_{104}, m_{108}; m_{60}, m_{68}, m_{108}, m_{112}; m_{64}, m_{66}, m_{112}, m_{116}; m_{64}, m_{72}, m_{116}, m_{120}; m_{68}, m_{70}, m_{120}, m_{124}; m_{68}, m_{76}, m_{124}, m_{128}; m_{72}, m_{74}, m_{128}, m_{132}; m_{72}, m_{80}, m_{132}, m_{136}; m_{76}, m_{78}, m_{136}, m_{140}; m_{76}, m_{84}, m_{140}, m_{144}; m_{80}, m_{82}, m_{144}, m_{148}; m_{80}, m_{88}, m_{148}, m_{152}; m_{84}, m_{86}, m_{152}, m_{156}; m_{84}, m_{92}, m_{156}, m_{160}; m_{88}, m_{90}, m_{160}, m_{164}; m_{88}, m_{96}, m_{164}, m_{168}; m_{92}, m_{94}, m_{168}, m_{172}; m_{92}, m_{100}, m_{172}, m_{176}; m_{96}, m_{98}, m_{176}, m_{180}; m_{96}, m_{104}, m_{180}, m_{184}; m_{100}, m_{102}, m_{184}, m_{188}; m_{100}, m_{108}, m_{188}, m_{192}; m_{104}, m_{106}, m_{192}, m_{196}; m_{104}, m_{112}, m_{196}, m_{200}; m_{108}, m_{110}, m_{200}, m_{204}; m_{108}, m_{116}, m_{204}, m_{208}; m_{112}, m_{114}, m_{208}, m_{212}; m_{112}, m_{120}, m_{212}, m_{216}; m_{116}, m_{118}, m_{216}, m_{220}; m_{116}, m_{124}, m_{220}, m_{224}; m_{120}, m_{122}, m_{224}, m_{228}; m_{120}, m_{128}, m_{228}, m_{232}; m_{124}, m_{126}, m_{232}, m_{236}; m_{124}, m_{132}, m_{236}, m_{240}; m_{128}, m_{130}, m_{240}, m_{244}; m_{128}, m_{136}, m_{244}, m_{248}; m_{132}, m_{134}, m_{248}, m_{252}; m_{132}, m_{140}, m_{252}, m_{256}; m_{136}, m_{138}, m_{256}, m_{260}; m_{136}, m_{144}, m_{260}, m_{264}; m_{140}, m_{142}, m_{264}, m_{268}; m_{140}, m_{148}, m_{268}, m_{272}; m_{144}, m_{146}, m_{272}, m_{276}; m_{144}, m_{152}, m_{276}, m_{280}; m_{148}, m_{150}, m_{280}, m_{284}; m_{148}, m_{156}, m_{284}, m_{288}; m_{152}, m_{154}, m_{288}, m_{292}; m_{152}, m_{160}, m_{292}, m_{296}; m_{156}, m_{158}, m_{296}, m_{300}; m_{156}, m_{164}, m_{300}, m_{304}; m_{160}, m_{162}, m_{304}, m_{308}; m_{160}, m_{168}, m_{308}, m_{312}; m_{164}, m_{166}, m_{312}, m_{316}; m_{164}, m_{172}, m_{316}, m_{320}; m_{168}, m_{170}, m_{320}, m_{324}; m_{168}, m_{176}, m_{324}, m_{328}; m_{172}, m_{174}, m_{328}, m_{332}; m_{172}, m_{180}, m_{332}, m_{336}; m_{176}, m_{178}, m_{336}, m_{340}; m_{176}, m_{184}, m_{340}, m_{344}; m_{180}, m_{182}, m_{344}, m_{348}; m_{180}, m_{188}, m_{348}, m_{352}; m_{184}, m_{186}, m_{352}, m_{356}; m_{184}, m_{192}, m_{356}, m_{360}; m_{188}, m_{190}, m_{360}, m_{364}; m_{188}, m_{196}, m_{364}, m_{368}; m_{192}, m_{194}, m_{368}, m_{372}; m_{192}, m_{200}, m_{372}, m_{376}; m_{196}, m_{198}, m_{376}, m_{380}; m_{196}, m_{204}, m_{380}, m_{384}; m_{200}, m_{202}, m_{384}, m_{388}; m_{200}, m_{208}, m_{388}, m_{392}; m_{204}, m_{206}, m_{392}, m_{396}; m_{204}, m_{212}, m_{396}, m_{400}; m_{208}, m_{210}, m_{400}, m_{404}; m_{208}, m_{216}, m_{404}, m_{408}; m_{212}, m_{214}, m_{408}, m_{412}; m_{212}, m_{220}, m_{412}, m_{416}; m_{216}, m_{218}, m_{416}, m_{420}; m_{216}, m_{224}, m_{420}, m_{424}; m_{220}, m_{222}, m_{424}, m_{428}; m_{220}, m_{228}, m_{428}, m_{432}; m_{224}, m_{226}, m_{432}, m_{436}; m_{224}, m_{232}, m_{436}, m_{440}; m_{228}, m_{230}, m_{440}, m_{444}; m_{228}, m_{236}, m_{444}, m_{448}; m_{232}, m_{234}, m_{448}, m_{452}; m_{232}, m_{240}, m_{452}, m_{456}; m_{236}, m_{238}, m_{456}, m_{460}; m_{236}, m_{244}, m_{460}, m_{464}; m_{240}, m_{242}, m_{464}, m_{468}; m_{240}, m_{248}, m_{468}, m_{472}; m_{244}, m_{246}, m_{472}, m_{476}; m_{244}, m_{252}, m_{476}, m_{480}; m_{248}, m_{250}, m_{480}, m_{484}; m_{248}, m_{256}, m_{484}, m_{488}; m_{252}, m_{254}, m_{488}, m_{492}; m_{252}, m_{260}, m_{492}, m_{496}; m_{256}, m_{258}, m_{496}, m_{500}; m_{256}, m_{264}, m_{500}, m_{504}; m_{260}, m_{262}, m_{504}, m_{508}; m_{260}, m_{268}, m_{508}, m_{512}; m_{264}, m_{266}, m_{512}, m_{516}; m_{264}, m_{272}, m_{516}, m_{520}; m_{268}, m_{270}, m_{520}, m_{524}; m_{268}, m_{276}, m_{524}, m_{528}; m_{272}, m_{274}, m_{528}, m_{532}; m_{272}, m_{280}, m_{532}, m_{536}; m_{276}, m_{278}, m_{536}, m_{540}; m_{276}, m_{284}, m_{540}, m_{544}; m_{280}, m_{282}, m_{544}, m_{548}; m_{280}, m_{288}, m_{548}, m_{552}; m_{284}, m_{286}, m_{552}, m_{556}; m_{284}, m_{292}, m_{556}, m_{560}; m_{288}, m_{290}, m_{560}, m_{564}; m_{288}, m_{296}, m_{564}, m_{568}; m_{292}, m_{294}, m_{568}, m_{572}; m_{292}, m_{300}, m_{572}, m_{576}; m_{296}, m_{298}, m_{576}, m_{580}; m_{296}, m_{304}, m_{580}, m_{584}; m_{300}, m_{302}, m_{584}, m_{588}; m_{300}, m_{308}, m_{588}, m_{592}; m_{304}, m_{306}, m_{592}, m_{596}; m_{304}, m_{312}, m_{596}, m_{600}; m_{308}, m_{310}, m_{600}, m_{604}; m_{308}, m_{316}, m_{604}, m_{608}; m_{312}, m_{314}, m_{608}, m_{612}; m_{312}, m_{320}, m_{612}, m_{616}; m_{316}, m_{318}, m_{616}, m_{620}; m_{316}, m_{324}, m_{620}, m_{624}; m_{320}, m_{322}, m_{624}, m_{628}; m_{320}, m_{328}, m_{628}, m_{632}; m_{324}, m_{326}, m_{632}, m_{636}; m_{324}, m_{332}, m_{636}, m_{640}; m_{328}, m_{330}, m_{640}, m_{644}; m_{328}, m_{336}, m_{644}, m_{648}; m_{332}, m_{334}, m_{648}, m_{652}; m_{332}, m_{340}, m_{652}, m_{656}; m_{336}, m_{338}, m_{656}, m_{660}; m_{336}, m_{344}, m_{660}, m_{664}; m_{340}, m_{342}, m_{664}, m_{668}; m_{340}, m_{348}, m_{668}, m_{672}; m_{344}, m_{346}, m_{672}, m_{676}; m_{344}, m_{352}, m_{676}, m_{680}; m_{348}, m_{350}, m_{680}, m_{684}; m_{348}, m_{356}, m_{684}, m_{688}; m_{352}, m_{354}, m_{688}, m_{692}; m_{352}, m_{360}, m_{692}, m_{696}; m_{356}, m_{358}, m_{696}, m_{700}; m_{356}, m_{364}, m_{700}, m_{704}; m_{360}, m_{362}, m_{704}, m_{708}; m_{360}, m_{368}, m_{708}, m_{712}; m_{364}, m_{366}, m_{712}, m_{716}; m_{364}, m_{372}, m_{716}, m_{720}; m_{368}, m_{370}, m_{720}, m_{724}; m_{368}, m_{376}, m_{724}, m_{728}; m_{372}, m_{374}, m_{728}, m_{732}; m_{372}, m_{380}, m_{732}, m_{736}; m_{376}, m_{378}, m_{736}, m_{740}; m_{376}, m_{384}, m_{740}, m_{744}; m_{380}, m_{382}, m_{744}, m_{748}; m_{380}, m_{388}, m_{748}, m_{752}; m_{384}, m_{386}, m_{752}, m_{756}; m_{384}, m_{392}, m_{756}, m_{760}; m_{388}, m_{390}, m_{760}, m_{764}; m_{388}, m_{396}, m_{764}, m_{768}; m_{392}, m_{394}, m_{768}, m_{772}; m_{392}, m_{400}, m_{772}, m_{776}; m_{396}, m_{398}, m_{776}, m_{780}; m_{396}, m_{404}, m_{780}, m_{784}; m_{400}, m_{402}, m_{784}, m_{788}; m_{400}, m_{408}, m_{788}, m_{792}; m_{404}, m_{406}, m_{792}, m_{796}; m_{404}, m_{412}, m_{796}, m_{800}; m_{408}, m_{410}, m_{800}, m_{804}; m_{408}, m_{416}, m_{804}, m_{808}; m_{412}, m_{414}, m_{808}, m_{812}; m_{412}, m_{420}, m_{812}, m_{816}; m_{416}, m_{418}, m_{816}, m_{820}; m_{416}, m_{424}, m_{820}, m_{824}; m_{420}, m_{422}, m_{824}, m_{828}; m_{420}, m_{428}, m_{828}, m_{832}; m_{424}, m_{426}, m_{832}, m_{836}; m_{424}, m_{432}, m_{836}, m_{840}; m_{428}, m_{430}, m_{840}, m_{844}; m_{428}, m_{436}, m_{844}, m_{848}; m_{432}, m_{434}, m_{848}, m_{852}; m_{432}, m_{440}, m_{852}, m_{856}; m_{436}, m_{438}, m_{856}, m_{860}; m_{436}, m_{444}, m_{860}, m_{864}; m_{440}, m_{442}, m_{864}, m_{868}; m_{440}, m_{448}, m_{868}, m_{872}; m_{444}, m_{446}, m_{872}, m_{876}; m_{444}, m_{452}, m_{876}, m_{880}; m_{448}, m_{450}, m_{880}, m_{884}; m_{448}, m_{456}, m_{884}, m_{888}; m_{452}, m_{454}, m_{888}, m_{892}; m_{452}, m_{460}, m_{892}, m_{896}; m_{456}, m_{458}, m_{896}, m_{900}; m_{456}, m_{464}, m_{900}, m_{904}; m_{460}, m_{462}, m_{904}, m_{908}; m_{460}, m_{468}, m_{908}, m_{912}; m_{464}, m_{466}, m_{912}, m_{916}; m_{464}, m_{472}, m_{916}, m_{920}; m_{468}, m_{470}, m_{920}, m_{924}; m_{468}, m_{476}, m_{924}, m_{928}; m_{472}, m_{474}, m_{928}, m_{932}; m_{472}, m_{480}, m_{932}, m_{936}; m_{476}, m_{478}, m_{936}, m_{940}; m_{476}, m_{484}, m_{940}, m_{944}; m_{480}, m_{482}, m_{944}, m_{948}; m_{480}, m_{488}, m_{948}, m_{952}; m_{484}, m_{486}, m_{952}, m_{956}; m_{484}, m_{492}, m_{956}, m_{960}; m_{488}, m_{490}, m_{960}, m_{964}; m_{488}, m_{496}, m_{964}, m_{968}; m_{492}, m_{494}, m_{968}, m_{972}; m_{492}, m_{500}, m_{972}, m_{976}; m_{496}, m_{498}, m_{976}, m_{980}; m_{496}, m_{504}, m_{980}, m_{984}; m_{500}, m_{502}, m_{984}, m_{988}; m_{500}, m_{508}, m_{988}, m_{992}; m_{504}, m_{506}, m_{992}, m_{996}; m_{504}, m_{512}, m_{996}, m_{1000}$.

The squares are read by dropping out the variables which change. Some possible

Grouping s is

(a) $m_0, m_{16} = \bar{B}\bar{C}\bar{D}\bar{E}$

$M_0, M_{16} = B + C + D + E$

(b) $m_2, m_{18} = \bar{B}\bar{C}D\bar{E}$

$M_2, M_{18} = B + C + \bar{D} + E$

(c) $m_4, m_6, m_{20}, m_{22} = \bar{B}C\bar{E}$

$M_4, M_6, M_{20}, M_{22} = B + \bar{C} + E$

(d) $m_5, m_7, m_{13}, m_{15}, m_{21}, m_{23},$

$M_5, M_7, M_{13}, M_{15}, M_{21}, M_{23}, M_{29},$

$m_{29}, m_{31} = CE$

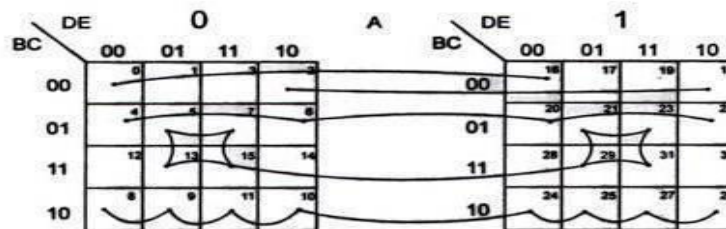
$M_{31} = \bar{C} + \bar{E}$

(e) $m_8, m_9, m_{10}, m_{11}, m_{24}, m_{25},$

$M_8, M_9, M_{10}, M_{11}, M_{24}, M_{25}, M_{26},$

$m_{26}, m_{27} = B\bar{C}$

$M_{27} = \bar{B} + C$



Ex: $F = \sum m(0,1,4,5,6,13,14,15,22,24,25,28,29,30,31)$ is SOP

POS is $F = \pi M(2,3,7,8,9,10,11,12,16,17,18,19,20,21,23,26,27)$

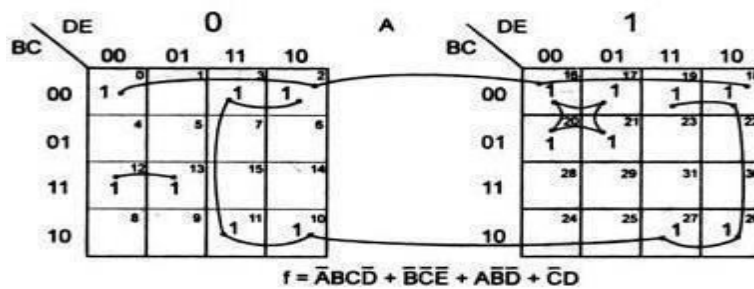
The real minimal expression is the minimal of the SOP and POS forms.

The reduction is done as

1. There is no isolated 1s
2. M_{12} can go only with m_{13} . Form a 2-square which is read as $A'B'CD'$
3. M_0 can go with m_2, m_{16} and m_{18} . so form a 4-square which is read as $B'C'E'$
4. M_{20}, m_{21}, m_{17} and m_{16} form a 4-square which is read as $AB'D'$
5. $M_2, m_3, m_{18}, m_{19}, m_{10}, m_{11}, m_{26}$ and m_{27} form an 8-square which is read as $C'D$
6. Write all the product terms in SOP form.

So the minimal expression is

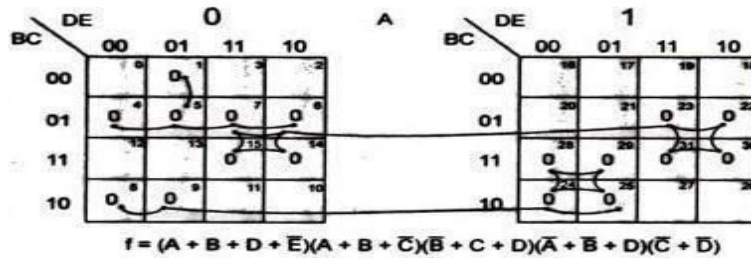
$$F_{\min} = A'B'CD' + B'C'E' + AB'D' + C'D \text{ (16 inputs)}$$



In the POS k-map ,the reduction is done as:

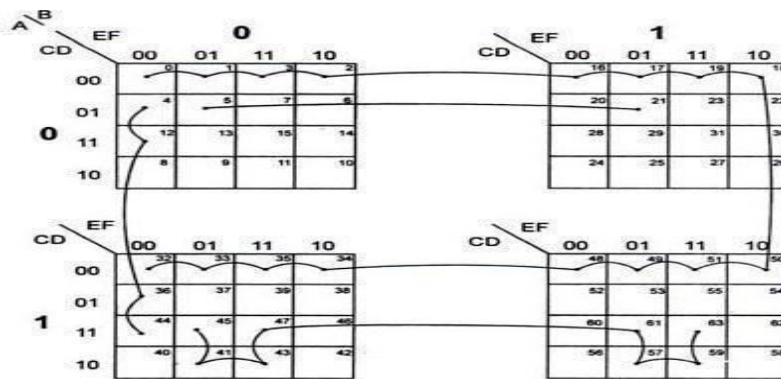
1. There are no isolated 0s
2. M_1 can go only with M_5 . So, make a 2-square, which is read as $(A + B + D + \bar{E})$.
3. M_4 can go with $M_5, M_7,$ and M_6 to form a 4-square, which is read as $(A + B + \bar{C})$.
4. M_8
5. M_{28}
6. M_{30}
7. Sum terms in POS form. So the minimal expression in POS is

$$F_{\min} = A'BcD' + B'C'E' + AB'D' + C'D$$



Six variable k-map:

Six variable k-map can have $2^6 = 64$ combinations as $ABCDEF$ with minterms m_0, m_1, \dots, m_{63} respectively in SOP & $(A+B+C+D+E+F)$, (\dots) with maxterms M_0, M_1, \dots, M_{63} respectively in POS form. It has $2^6 = 64$ squares or cells of the k-map are divided into 4 blocks of 16 squares each.



Some possible groupings in a six variable k-map

Don't care combinations: For certain input combinations, the value of the output is unspecified either because the input combinations are invalid or because the precise value of the output is of no consequence. The combinations for which the value of experiments are not specified are called don't care combinations are invalid or because the precise value of the output is of no consequence. The combinations for which the value of expressions is not specified are called don't care combinations or Optional Combinations, such expressions stand incompletely specified. The output is a don't care for these invalid combinations.

Ex: In XS-3 code system, the binary states 0000, 0001, 0010, 1101, 1110, 1111 are unspecified. & never occur called don't cares.

A standard SOP expression with don't cares can be converted into a standard POS form by keeping the don't cares as they are & writing the missing minterms of the SOP form as the maxterms of the POS form viceversa.

Don't cares denoted by $_X'$ or $_\phi'$

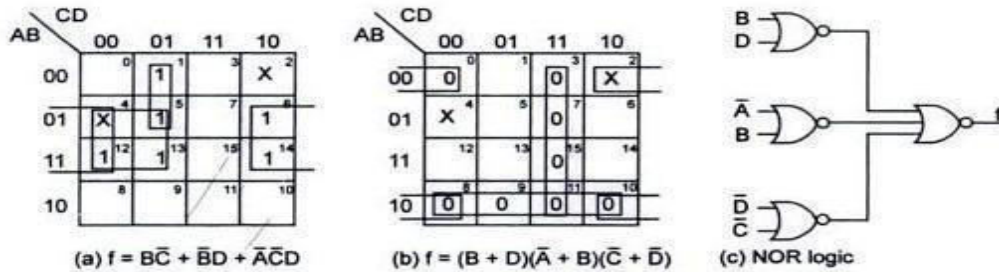
Ex: $f = \sum m(1,5,6,12,13,14) + d(2,4)$

Or $f = \pi M(0,3,7,9,10,11,15) \cdot \pi d(2,4)$

SOP minimal form $f_{min} = \dots + B + \dots$

POS minimal form $f_{min} = (B+D)(\dots + B)(\dots + D)$

$= \dots + \dots + \dots + \dots + (\dots + \dots)$



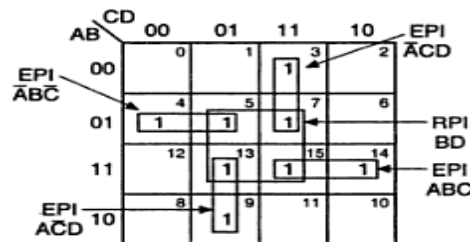
Prime implicants, Essential Prime implicants, Redundant prime implicants:

Each square or rectangle made up of the bunch of adjacent minterms is called a subcube. Each of these subcubes is called a Prime implicant (PI). The PI which contains at least one minterm which cannot be covered by any other prime implicants is called as Essential Prime implicant (EPI). The PI whose each 1 is covered at least by one EPI is called a Redundant Prime implicant (RPI). A PI which is neither an EPI nor a RPI is called a Selective Prime implicant (SPI).

The function has unique MSP comprising EPI is

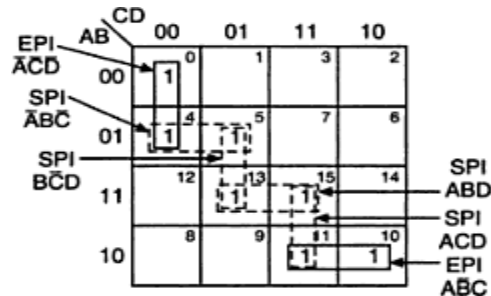
$F(A,B,C,D) = CD + ABC + A\bar{D} + B$

The RPI $\bar{B}D$ may be included without changing the function but the resulting expression would not be in minimal SOP(MSP) form.



Essential and Redundant Prime Implicants

$F(A,B,C,D)=\sum m(0,4,5,10,11,13,15)$ SPI are marked by dotted squares, shows MSP form of a function need not be unique.



Essential and Selective Prime Implicants

Here, the MSP form is obtained by including two EPI's & selecting a set of SPI's to cover remaining uncovered minterms 5,13,15. & these can be covered as

(A) (4,5) & (13,15) ----- $B + ABD$

(B) (5,13) & (13,15) ----- $B D + ABD$

(C) (5,13) & (15,11) ----- $B D + ACD$

$F(A,B,C,D) = +A C$ ----- EPI's + $B + ABD$

(OR) $F(A,B,C,D) = +A C$ ----- EPI's + $B D + ABD$

(OR) $F(A,B,C,D) = +A C$ ----- EPI's + $B D + ACD$

False PI's Essential False PI's, Redundant False PI's & Selective False PI's:

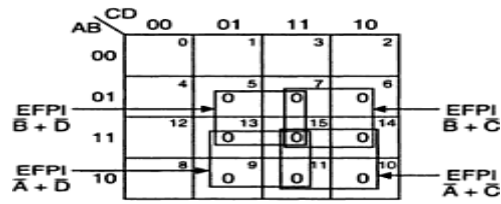
The maxterms are called false minterms. The PI's is obtained by using the maxterms are called False PI's (FPI). The FPI which contains at least one '0' which can't be covered by only other FPI is called an Essential False Prime implicant (ESPI)

$F(A,B,C,D) = \sum m(0,1,2,3,4,8,12)$

$= \pi M(5,6,7,9,10,11,13,14,15)$

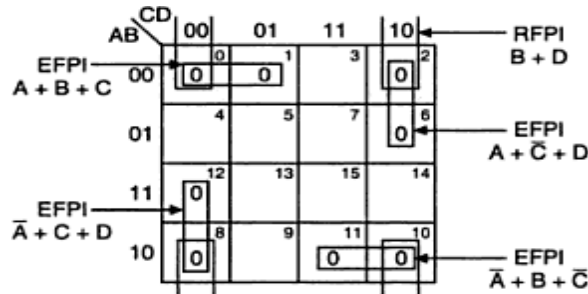
$F_{min} = (+)(+)(+)(+)$

All the FPI, EFPI's as each of them contain atleast one '0' which can't be covered by any other FPI



Essential False Prime implicants

Consider Function $F(A,B,C,D) = \pi M(0,1,2,6,8,10,11,12)$



Essential and Redundant False Prime Implicants

Mapping when the function is not expressed in minterms (maxterms):

An expression in k-map must be available as a sum (product) of minterms (maxterms). However if not so expressed, it is not necessary to expand the expression algebraically into its minterms (maxterms). Instead, expansion into minterms (maxterms) can be accomplished in the process of entering the terms of the expression on the k-map.

Limitations of Karnaugh maps:

- Convenient as long as the number of variables does not exceed six.
- Manual technique, simplification process is heavily dependent on the human abilities.

Quine-Mccluskey Method:

It also known as *Tabular method*. It is more systematic method of minimizing expressions of even larger number of variables. It is suitable for hand computation as well as computation by machines i.e., programmable. . The procedure is based on repeated application of the combining theorem.

$PA+P = P$ (P is set of literals) on all adjacent pairs of terms, yields the set of all PI's from which a minimal sum may be selected.

Consider expression

$$\sum m(0,1,4,5) = + C + A + A C$$

First, second terms & third, fourth terms can be combined

$$(+) + (C+) = +A$$

Reduced to

$$(+) =$$

The same result can be obtained by combining m_0 & m_4 & m_1 & m_5 in first step & resulting terms in the second step .

Procedure:

- Decimal Representation
- Don't cares
- PI chart
- EPI
- Dominating Rows & Columns
- Determination of Minimal expressions in complex cases.

Branching Method:

EXAMPLE 3.29 Obtain the set of prime implicants for the Boolean expression $f = \Sigma m(0, 1, 6, 7, 8, 9, 13, 14, 15)$ using the tabular method.

Solution

Group the minterms in terms of the number of 1s present in them and write their binary designations. The procedure to obtain the prime implicants is shown in Table 3.3.

Table 3.3 Example 3.29

	Column 1		Column 2		Column 3
	Minterm	Binary designation	A	B C D	A B C D
Index 0	0	0000 ✓	0, 1 (1)	0 0 0 - ✓	0, 1, 8, 9 (1, 8) - 00 - Q
Index 1	1	0001 ✓	0, 8 (8)	- 0 0 0 ✓
	8	1000 ✓	1, 9 (8)	- 0 0 1 ✓
Index 2	6	0110 ✓	8, 9 (1)	1 0 0 - ✓	6, 7, 14, 15 (1, 8) - 11 - P
	9	1001 ✓	6, 7 (1)	0 1 1 - ✓	
Index 3	7	0111 ✓	6, 14 (8)	- 1 1 0 ✓	
	13	1101 ✓	9, 13 (4)	1 - 0 1 S	
	14	1110 ✓	7, 15 (8)	- 1 1 1 ✓	
Index 4	15	1111 ✓	13, 15 (2)	1 1 - 1 R	
			14, 15 (1)	1 1 1 - ✓	

Comparing the terms of index 0 with the terms of index 1 of column 1, $m_0(0000)$ is combined with $m_1(0001)$ to yield 0, 1 (1), i.e. 000–. This is recorded in column 2 and 0000 and 0001 are checked off in column 1. $m_0(0000)$ is combined with $m_8(1000)$ to yield 0, 8 (8), i.e. –000. This is recorded in column 2 and 1000 is checked off in column 1. Note that 0000 of column 1 has already been checked off. No more combinations of terms of index 0 and index 1 are possible. So, draw a line below the last combination of these groups, i.e. below 0, 8 (8), –000 in column 2. Now 0, 1 (1), i.e. 000– and 0, 8 (8), i.e. –000 are the terms in the first group of column 2.

Comparing the terms of index 1 with the terms of index 2 in column 1, $m_1(0001)$ is combined with $m_9(1001)$ to yield 1, 9 (8), i.e. –001. This is recorded in column 2 and 1001 is checked off in column 1 because 0001 has already been checked off. $m_8(1000)$ is combined with $m_9(1001)$ to yield 8, 9 (1), i.e. 100–. This is recorded in column 2. 1000 and 1001 of column 1 have already been checked off. So, no need to check them off again. No more combinations of terms of index 1 and index 2 are possible. So, draw a line below the last combination of these groups, i.e. 8, 9 (1),

–001 in column 2. Now 1, 9 (8), i.e. –001 and 8, 9 (1), i.e. 100– are the terms in the second group of column 2.

Similarly, comparing the terms of index 2 with the terms of index 3 in column 1,

$m_6(0110)$ and $m_7(0111)$ yield 6, 7 (1), i.e. 011–. Record it in column 2 and check off 6(0110) and 7(0111).

$m_6(0110)$ and $m_{14}(1110)$ yield 6, 14 (8), i.e. –110. Record it in column 2 and check off 6(0110) and 14(1110).

$m_9(1001)$ and $m_{13}(1101)$ yield 9, 13 (4), i.e. 1–01. Record it in column 2 and check off 9(1001) and 13(1101).

So, 6, 7 (1), i.e. 011–, and 6, 14 (8), i.e. –110 and 9, 13 (4), i.e. 1–01 are the terms in group 3 of column 2. Draw a line at the end of 9, 13 (4), i.e. 1–01.

Also, comparing the terms of index 3 with the terms of index 4 in column 1,

$m_7(0111)$ and $m_{15}(1111)$ yield 7, 15 (8), i.e. –111. Record it in column 2 and check off 7(0111) and 15(1111).

$m_{13}(1101)$ and $m_{15}(1111)$ yield 13, 15 (2), i.e. 11–1. Record it in column 2 and check off 13 and 15.

$m_{14}(1110)$ and $m_{15}(1111)$ yield 14, 15 (1), i.e. 111–. Record it in column 2 and check off 14 and 15.

So, 7, 15 (8), i.e. –111, and 13, 15 (2), i.e. 11–1 and 14, 15 (1), i.e. 111– are the terms in group 4 of column 2. Column 2 is completed now.

Comparing the terms of group 1 with the terms of group 2 in column 2, the terms 0, 1 (1), i.e. 000– and 8, 9 (1), i.e. 100– are combined to form 0, 1, 8, 9 (1, 8), i.e. –00–. Record it in group 1 of column 3 and check off 0, 1 (1), i.e. 000–, and 8, 9 (1), i.e. 100– of column 2. The terms 0, 8 (8), i.e. –000 and 1, 9 (8), i.e. –001 are combined to form 0, 1, 8, 9 (1, 8), i.e. –00–. This has already been recorded in column 3. So, no need to record again. Check off 0, 8 (8), i.e. –000 and 1, 9 (8), i.e. –001 of column 2. Draw a line below 0, 1, 8, 9 (1, 8), i.e. –00–. This is the only term in group 1 of column 3. No term of group 2 of column 2 can be combined with any term of group 3 of column 2. So, no entries are made in group 2 of column 2.

Comparing the terms of group 3 of column 2 with the terms of group 4 of column 2, the terms 6, 7 (1), i.e. 011–, and 14, 15 (1), i.e. 111– are combined to form 6, 7, 14, 15 (1, 8), i.e. –11–. Record it in group 3 of column 3 and check off 6, 7 (1), i.e. 011– and 14, 15 (1), i.e. 111– of column 2. The terms 6, 14 (8), i.e. –110 and 7, 15 (8), i.e. –111 are combined to form 6, 7, 14, 15 (1, 8), i.e. –11–. This has already been recorded in column 3; so, check off 6, 14 (8), i.e. –110 and 7, 15 (8), i.e. –111 of column 2.

Observe that the terms 9, 13 (4), i.e. 1–01 and 13, 15 (2), i.e. 11–1 cannot be combined with any other terms. Similarly in column 3, the terms 0, 1, 8, 9 (1, 8), i.e. –00– and 6, 7, 14, 15 (1, 8), i.e. –11– cannot also be combined with any other terms. So, these 4 terms are the prime implicants.

The terms, which cannot be combined further, are labelled as P, Q, R, and S. These form the set of prime implicants.

EX:

Obtain the minimal expression for $f = \sum m(1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$ using the tabular method.

Solution

The procedure to obtain the set of prime implicants is illustrated in Table 3.4.

Table 3.4 Example 3.30

	Step 1	Step 2	Step 3	
Index 1	1 ✓	1, 3 (2) ✓	1, 3, 5, 7 (2, 4)	T
	2 ✓	1, 5 (4) ✓	1, 5, 9, 13 (4, 8)	S
	8 ✓	1, 9 (8) ✓	2, 3, 6, 7 (1, 4)	R
Index 2	3 ✓	2, 3 (1) ✓	8, 9, 12, 13 (1, 4)	Q
	5 ✓	2, 6 (4) ✓	5, 7, 13, 15 (2, 8)	P
	6 ✓	8, 9 (1) ✓		
	9 ✓	8, 12 (4) ✓		
Index 3	12 ✓	3, 7 (4) ✓		
	7 ✓	5, 7 (2) ✓		
Index 3	13 ✓	5, 13 (8) ✓		
	Index 4	15 ✓	6, 7 (1) ✓	
		9, 13 (4) ✓		
		12, 13 (1) ✓		
		7, 15 (8) ✓		
		13, 15 (2) ✓		

The non-combinable terms P, Q, R, S and T are recorded as prime implicants.

$$P \rightarrow 5, 7, 13, 15 (2, 8) = X 1 X 1 = BD$$

(Literals with weights 2 and 8, i.e. C and A are deleted. The lowest minterm is $m_5(5 = 4 + 1)$. So, literals with weights 4 and 1, i.e. B and D are present in non-complemented form. So, read it as BD.)

$$Q \rightarrow 8, 9, 12, 13 (1, 4) = 1 X 0 X = A\bar{C}$$

(Literals with weights 1 and 4, i.e. D and B are deleted. The lowest minterm is m_8 . So, literal with weight 8 is present in non-complemented form and literal with weight 2 is present in complemented form. So, read it as $A\bar{C}$.)

$$R \rightarrow 2, 3, 6, 7 (1, 4) = 0 X 1 X = \bar{A}C$$

(Literals with weights 1 and 4, i.e. D and B are deleted. The lowest minterm is m_2 . So, literal with weight 2 is present in non-complemented form and literal with weight 8 is present in complemented form. So, read it as $\bar{A}C$.)

$$S \rightarrow 1, 5, 9, 13 (4, 8) = X X 0 1 = \bar{C}D$$

(Literals with weights 4 and 8, i.e. B and A are deleted. The lowest minterm is m_1 . So, literal with weight 1 is present in non-complemented form and literal with weight 2 is present in complemented form. So, read it as $\bar{C}D$.)

$$T \rightarrow 1, 3, 5, 7 (2, 4) = 0 X X 1 = \bar{A}D$$

(Literals with weights 2 and 4, i.e. C and B are deleted. The lowest minterm is 1. So, literal with weight 1 is present in non-complemented form and literal with weight 8 is present in complemented form. So, read it as $\bar{A}D$.)

The prime implicant chart of the expression

$$f = \sum m(1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$$

is as shown in Table 3.5. It consists of 11 columns corresponding to the number of minterms and 5 rows corresponding to the prime implicants P, Q, R, S, and T generated. Row R contains four \times s at the intersections with columns 2, 3, 6, and 7, because these minterms are covered by the prime implicant R. A row is said to cover the columns in which it has \times s. The problem now is to select a minimal subset of prime implicants, such that each column contains at least one \times in the rows corresponding to the selected subset and the total number of literals in the prime implicants selected is as small as possible. These requirements guarantee that the number of unions of the selected prime implicants is equal to the original number of minterms and that, no other expression containing fewer literals can be found.

Table 3.5 Example 3.30: Prime implicant chart

	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	
	1	2	3	5	6	7	8	9	12	13	15
*P $\rightarrow 5, 7, 13, 15 (2, 8)$				\times		\times				\times	\times
*Q $\rightarrow 8, 9, 12, 13 (1, 4)$							\times	\times	\times	\times	
*R $\rightarrow 2, 3, 6, 7 (1, 4)$		\times	\times		\times	\times					
S $\rightarrow 1, 5, 9, 13 (4, 8)$	\times			\times				\times		\times	
T $\rightarrow 1, 3, 5, 7 (2, 4)$	\times		\times	\times		\times					

In the prime implicant chart of Table 3.5, m_2 and m_6 are covered by R only. So, R is an essential prime implicant. So, check off all the minterms covered by it, i.e. m_2 , m_3 , m_6 , and m_7 . Q is also an essential prime implicant because only Q covers m_8 and m_{12} . Check off all the minterms covered by it, i.e. m_8 , m_9 , m_{12} , and m_{13} . P is also an essential prime implicant, because m_{15} is covered only by P. So check off m_{15} , m_5 , m_7 , and m_{13} covered by it. Thus, only minterm 1 is not covered. Either row S or row T can cover it and both have the same number of literals. Thus, two minimal expressions are possible.

$$P + Q + R + S = BD + A\bar{C} + \bar{A}C + \bar{C}D$$

or

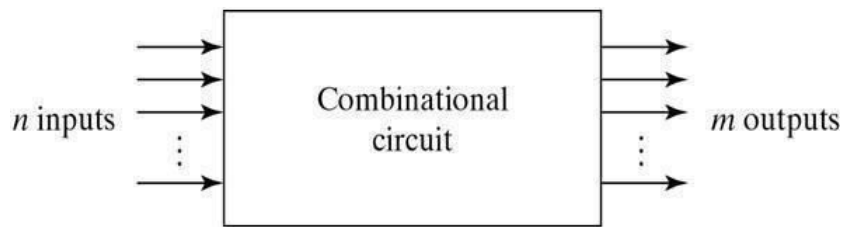
$$P + Q + R + T = BD + A\bar{C} + \bar{A}C + \bar{A}D$$

UNIT-II

COMBINATIONAL LOGIC CIRCUITS

Combinational Logic

- Logic circuits for digital systems may be combinational or sequential.
- A combinational circuit consists of input variables, logic gates, and output variables.



For n input variables, there are 2^n possible combinations of binary input variables. For each possible input combination, there is one and only one possible output combination. A combinational circuit can be described by m Boolean functions, one for each output variable. Usually the inputs come from flip-flops and outputs go to flip-flops.

Design Procedure:

1. The problem is stated
2. The number of available input variables and required output variables is determined.
3. The input and output variables are assigned letters/symbols.
4. The truth table that defines the required relationship between inputs and outputs is derived.
5. The simplified Boolean function for each output is obtained.

Adders:

Digital computers perform a variety of information processing tasks, the one is arithmetic operations. And the most basic arithmetic operation is the addition of two binary digits. i.e., 4 basic possible operations are:

$$0+0=0, 0+1=1, 1+0=1, 1+1=10$$

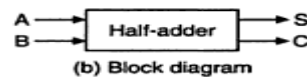
The first three operations produce a sum whose length is one digit, but when augends and addend bits are equal to 1, the binary sum consists of two digits. The higher significant bit of this result is called a carry. A combinational circuit that performs the addition of two bits is called a half-

adder. One that performs the addition of 3 bits (two significant bits & previous carry) is called a full adder. & 2 half adder can employ as a full-adder.

The Half Adder: A Half Adder is a combinational circuit with two binary inputs (augends and addend bits and two binary outputs (sum and carry bits.) It adds the two inputs (A and B) and produces the sum (S) and the carry (C) bits. It is an arithmetic operation of addition of two single bit words.

Inputs		Outputs	
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

(a) Truth table



(b) Block diagram

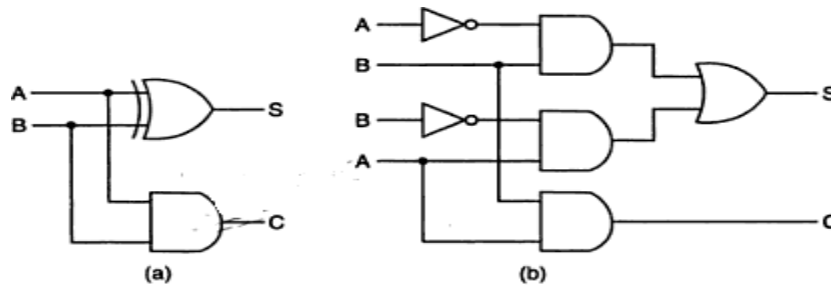
The Sum(S) bit and the carry (C) bit, according to the rules of binary addition, the sum (S) is the X-OR of A and B (It represents the LSB of the sum). Therefore,

$$S = AB + A \oplus B$$

The carry (C) is the AND of A and B (it is 0 unless both the inputs are 1). Therefore,

$$C = AB$$

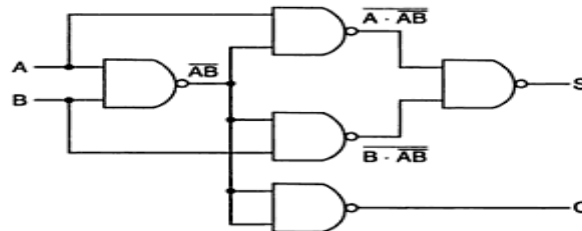
A half-adder can be realized by using one X-OR gate and one AND gate a



Logic diagrams of half-adder

NAND LOGIC:

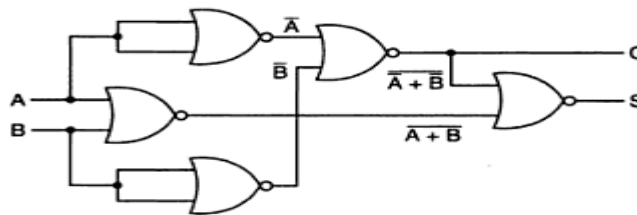
$$\begin{aligned}
 S &= A\bar{B} + \bar{A}B = A\bar{B} + A\bar{A} + \bar{A}B + B\bar{B} \\
 &= A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B}) \\
 &= A \cdot \bar{A}\bar{B} + B \cdot \bar{A}\bar{B} \\
 &= \overline{A \cdot \bar{A}\bar{B} \cdot B \cdot \bar{A}\bar{B}} \\
 C &= AB = \overline{\bar{A}\bar{B}}
 \end{aligned}$$



Logic diagram of a half-adder using only 2-input NAND gates.

NOR Logic:

$$\begin{aligned}
 S &= A\bar{B} + \bar{A}B = A\bar{B} + A\bar{A} + \bar{A}B + B\bar{B} \\
 &= A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B}) \\
 &= (A + B)(\bar{A} + \bar{B}) \\
 &= \overline{A + B + \bar{A} + \bar{B}} \\
 C &= AB = \overline{\bar{A}\bar{B}} = \overline{A + B}
 \end{aligned}$$



Logic diagram of a half-adder using only 2-input NOR gates.

The Full Adder:

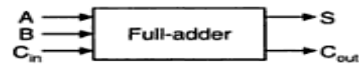
A Full-adder is a combinational circuit that adds two bits and a carry and outputs a sum bit and a carry bit. To add two binary numbers, each having two or more bits, the LSBs can be added by using a half-adder. The carry resulted from the addition of the LSBs is carried over to the next significant column and added to the two bits in that column. So, in the second and higher columns, the two data bits of that column and the carry bit generated from the addition in the previous column need to be added.

The full-adder adds the bits A and B and the carry from the previous column called the carry-in C_{in} and outputs the sum bit S and the carry bit called the carry-out C_{out} . The variable S gives the value of the least significant bit of the sum. The variable C_{out} gives the output carry. The

eight rows under the input variables designate all possible combinations of 1s and 0s that these variables may have. The 1s and 0s for the output variables are determined from the arithmetic sum of the input bits. When all the bits are 0s, the output is 0. The S output is equal to 1 when only 1 input is equal to 1 or when all the inputs are equal to 1. The C_{out} has a carry of 1 if two or three inputs are equal to 1.

Inputs			Sum	Carry
A	B	C _{in}	S	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

(a) Truth table



(b) Block diagram

Full-adder.

From the truth table, a circuit that will produce the correct sum and carry bits in response to every possible combination of A,B and C_{in} is described by

$$S = \overline{A}BC_{in} + A\overline{B}C_{in} + AB\overline{C_{in}} + ABC_{in}$$

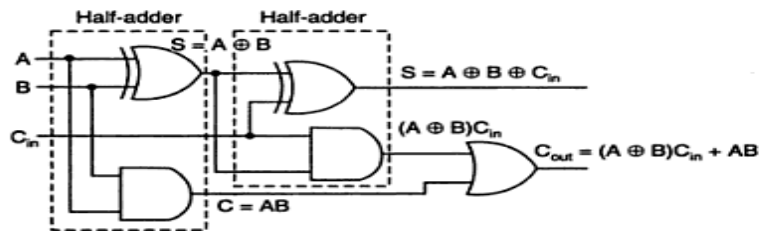
$$C_{out} = \overline{A}BC_{in} + A\overline{B}C_{in} + AB\overline{C_{in}} + ABC_{in}$$

and

$$S = A \oplus B \oplus C_{in}$$

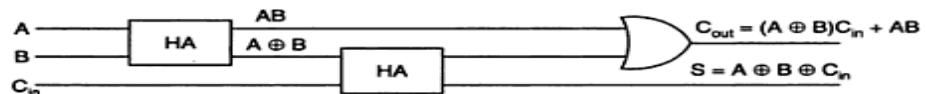
$$C_{out} = AC_{in} + BC_{in} + AB$$

The sum term of the full-adder is the X-OR of A,B, and C_{in}, i.e, the sum bit the modulo sum of the data bits in that column and the carry from the previous column. The logic diagram of the full-adder using two X-OR gates and two AND gates (i.e, Two half adders) and one OR gate is



Logic diagram of a full-adder using two half-adders.

The block diagram of a full-adder using two half-adders is :



Block diagram of a full-adder using two half-adders.

Even though a full-adder can be constructed using two half-adders, the disadvantage is that the bits must propagate through several gates in accession, which makes the total propagation delay greater than that of the full-adder circuit using AOI logic.

The Full-adder neither can also be realized using universal logic, i.e., either only NAND gates or only NOR gates as

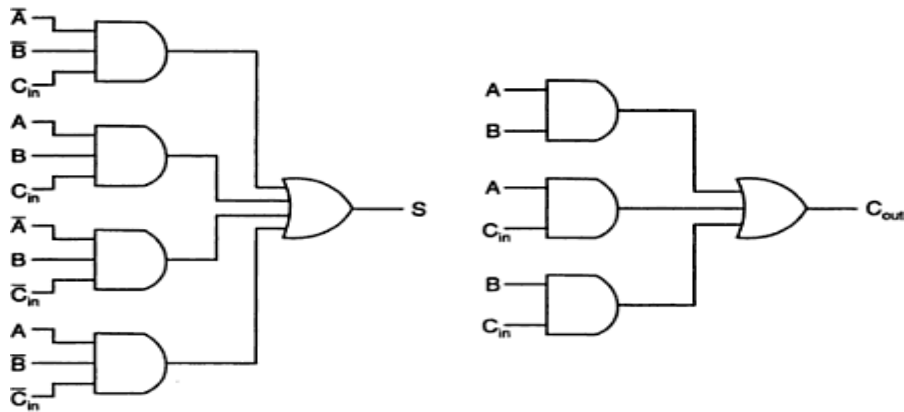
$$A \oplus B = \overline{\overline{A \cdot \overline{AB}} \cdot \overline{B \cdot \overline{AB}}}$$

Then

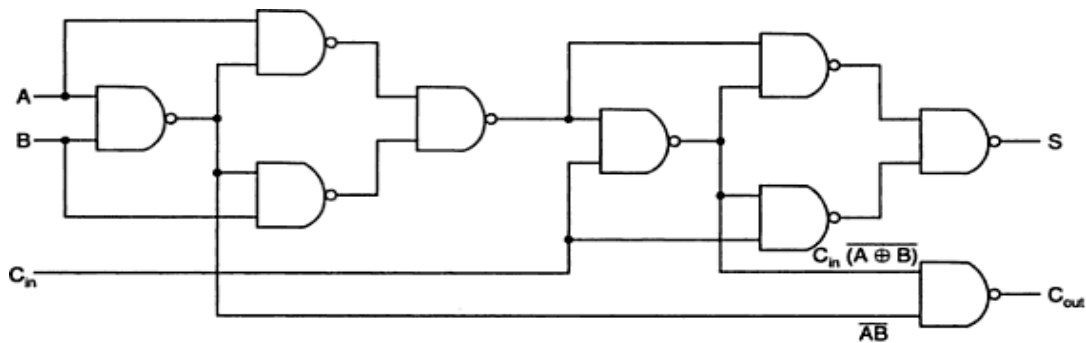
$$S = A \oplus B \oplus C_{in} = \overline{\overline{(A \oplus B) \cdot (A \oplus B)C_{in}} \cdot \overline{C_{in} \cdot (A \oplus B)C_{in}}}$$

NAND Logic:

$$C_{out} = C_{in}(A \oplus B) + AB = \overline{\overline{C_{in}(A \oplus B)} \cdot \overline{AB}}$$



Sum and carry bits of a full-adder using AOI logic.



Logic diagram of a full-adder using only 2-input NAND gates.

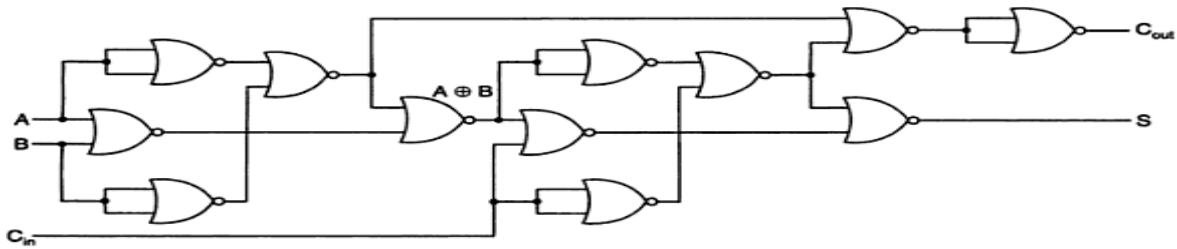
NOR Logic:

Then

$$A \oplus B = \overline{\overline{A + B} + \overline{A} + \overline{B}}$$

$$S = A \oplus B \oplus C_{in} = \overline{\overline{A \oplus B} + \overline{C_{in}} + \overline{A \oplus B} + \overline{C_{in}}}$$

$$C_{out} = AB + C_{in}(A \oplus B) = \overline{\overline{A} + \overline{B} + \overline{C_{in}} + A \oplus B}$$



Logic diagram of a full-adder using only 2-input NOR gates.

Subtractors:

The subtraction of two binary numbers may be accomplished by taking the complement of the subtrahend and adding it to the minuend. By this, the subtraction operation becomes an addition operation and instead of having a separate circuit for subtraction, the adder itself can be used to perform subtraction. This results in reduction of hardware. In subtraction, each subtrahend bit of the number is subtracted from its corresponding significant minuend bit to form a difference bit. If the minuend bit is smaller than the subtrahend bit, a 1 is borrowed from the next significant position., that has been borrowed must be conveyed to the next higher pair of bits by means of a signal coming out (output) of a given stage and going into (input) the next higher stage.

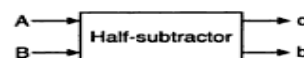
The Half-Subtractor:

A Half-subtractor is a combinational circuit that subtracts one bit from the other and produces the difference. It also has an output to specify if a 1 has been borrowed. . It is used to subtract the LSB of the subtrahend from the LSB of the minuend when one binary number is subtracted from the other.

A Half-subtractor is a combinational circuit with two inputs A and B and two outputs d and b. d indicates the difference and b is the output signal generated that informs the next stage that a 1 has been borrowed. When a bit B is subtracted from another bit A, a difference bit (d) and a borrow bit (b) result according to the rules given as

Inputs		Outputs	
A	B	d	b
0	0	0	0
1	0	1	0
1	1	0	0
0	1	1	1

(a) Truth table



(b) Block diagram

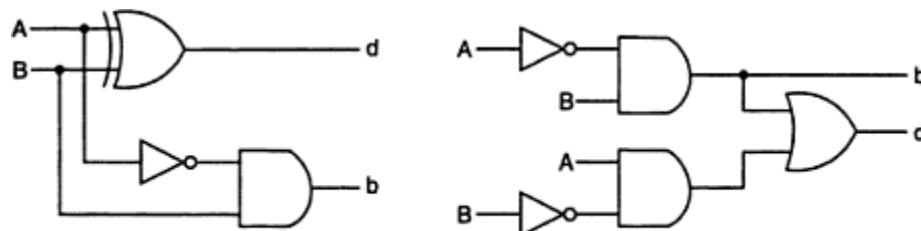
Half-subtractor.

The output borrow b is a 0 as long as $A \geq B$. It is a 1 for $A=0$ and $B=1$. The d output is the result of the arithmetic operation $2b+A-B$.

A circuit that produces the correct difference and borrow bits in response to every possible combination of the two 1-bit numbers is, therefore,

$$d = A \oplus B + A \bar{B} \quad \text{and} \quad b = A \bar{B}$$

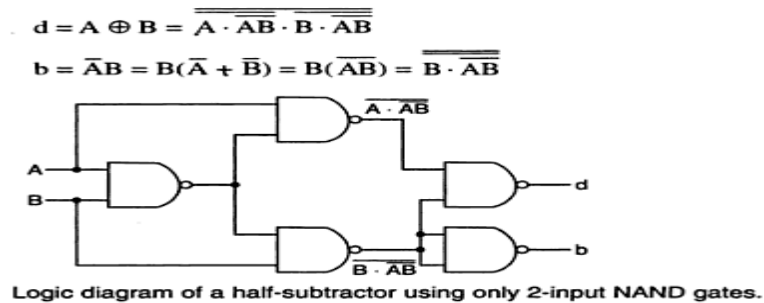
That is, the difference bit is obtained by X-OR ing the two inputs, and the borrow bit is obtained by ANDing the complement of the minuend with the subtrahend. Note that logic for this exactly the same as the logic for output S in the half-adder.



Logic diagrams of a half-subtractor.

A half-subtractor can also be realized using universal logic either using only NAND gates or using NOR gates as:

NAND Logic:

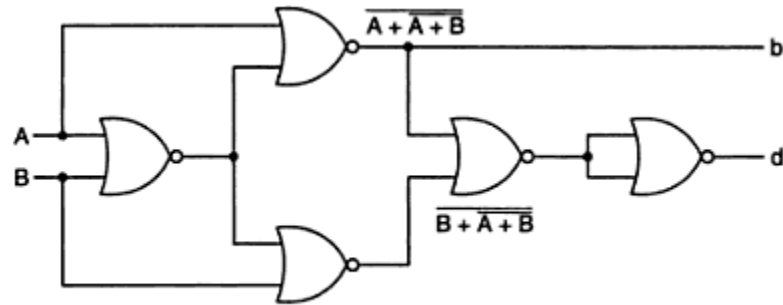


NOR Logic:

$$d = A \oplus B = A\bar{B} + \bar{A}B = A\bar{B} + B\bar{B} + \bar{A}B + A\bar{A}$$

$$= \bar{B}(A+B) + \bar{A}(A+B) = \overline{B+A+B} + \overline{A+A+B}$$

$$d = \bar{A}B = \bar{A}(A+B) = \overline{\bar{A}(A+B)} = A + (A+B)$$



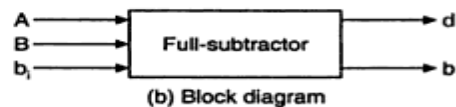
Logic diagram of a half-subtractor using only 2-input NOR gates.

The Full-Subtractor:

The half-subtractor can be only for LSB subtraction. IF there is a borrow during the subtraction of the LSBs, it affects the subtraction in the next higher column; the subtrahend bit is subtracted from the minuend bit, considering the borrow from that column used for the subtraction in the preceding column. Such a subtraction is performed by a full-subtractor. It subtracts one bit (B) from another bit (A), when already there is a borrow b_i from this column for the subtraction in the preceding column, and outputs the difference bit (d) and the borrow bit (b) required from the next d and b. The two outputs present the difference and output borrow. The 1s and 0s for the output variables are determined from the subtraction of $A-B-b_i$.

Inputs			Difference	Borrow
A	B	b_i	d	b
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

(a) Truth table



(b) Block diagram

Full-subtractor.

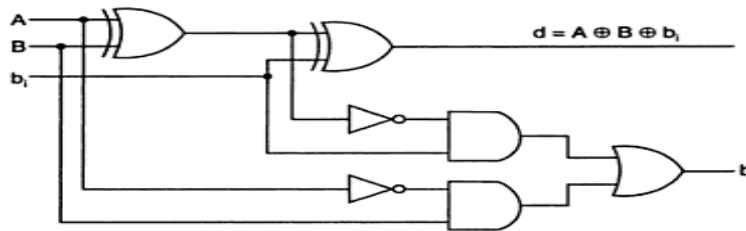
From the truth table, a circuit that will produce the correct difference and borrow bits in response to every possible combinations of A, B and b_i is

$$\begin{aligned}
 d &= \overline{A}Bb_i + \overline{A}B\overline{b_i} + A\overline{B}b_i + ABb_i \\
 &= b_i(AB + \overline{A}B) + \overline{b_i}(A\overline{B} + \overline{A}B) \\
 &= b_i(\overline{A \oplus B}) + \overline{b_i}(A \oplus B) = A \oplus B \oplus b_i
 \end{aligned}$$

and

$$\begin{aligned}
 b &= \overline{A}Bb_i + \overline{A}B\overline{b_i} + \overline{A}Bb_i + ABb_i = \overline{A}B(b_i + \overline{b_i}) + (AB + \overline{A}B)b_i \\
 &= \overline{A}B + (A \oplus B)b_i
 \end{aligned}$$

A full-subtractor can be realized using X-OR gates and AOI gates as

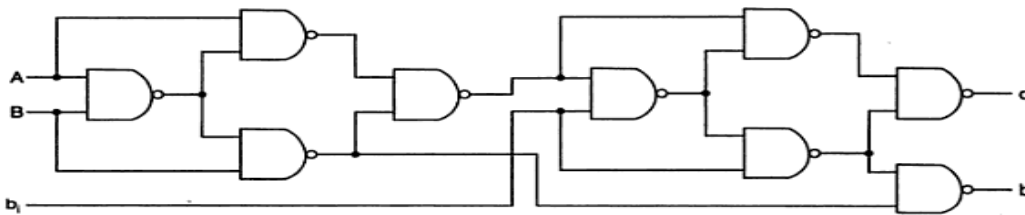


Logic diagram of a full-subtractor.

The full subtractor can also be realized using universal logic either using only NAND gates or using NOR gates as:

NAND Logic:

$$\begin{aligned}
 d &= A \oplus B \oplus b_i = \overline{\overline{(A \oplus B)} \oplus b_i} = \overline{\overline{(A \oplus B)}(A \oplus B)b_i \cdot b_i(A \oplus B)b_i} \\
 b &= \overline{AB} + b_i(\overline{A \oplus B}) = \overline{AB + b_i(A \oplus B)} \\
 &= \overline{AB \cdot b_i(A \oplus B)} = \overline{B(\overline{A + B}) \cdot b_i(b_i + (A \oplus B))} \\
 &= \overline{B \cdot AB \cdot b_i[b_i \cdot (A \oplus B)]}
 \end{aligned}$$

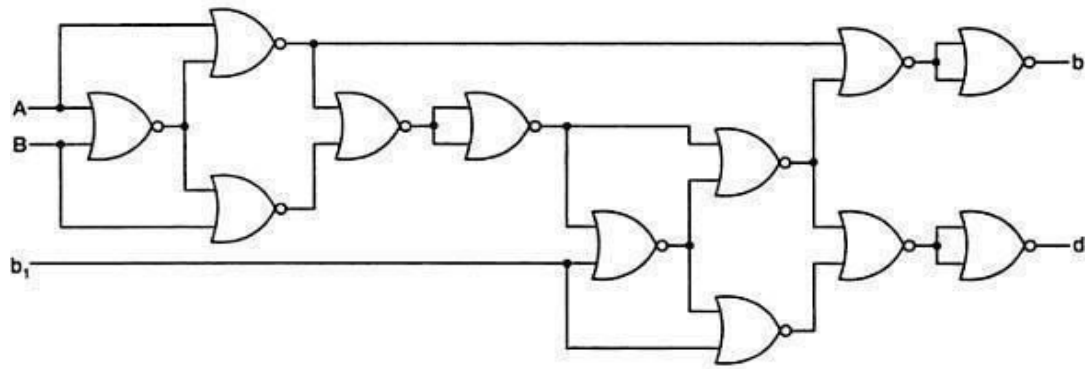


Logic diagram of a full-subtractor using only 2-input NAND gates.

NOR Logic:

$$\begin{aligned}
 d &= A \oplus B \oplus b_i = \overline{\overline{(A \oplus B)} \oplus b_i} \\
 &= \overline{(A \oplus B)b_i + (A \oplus B)\overline{b_i}} \\
 &= \overline{[(A \oplus B) + (A \oplus B)\overline{b_i}][b_i + (A \oplus B)\overline{b_i}]} \\
 &= \overline{(A \oplus B) + (A \oplus B) + b_i + b_i + (A \oplus B) + b_i} \\
 &= \overline{(A \oplus B) + (A \oplus B) + b_i + b_i + (A \oplus B) + b_i} \\
 b &= \overline{AB} + b_i(\overline{A \oplus B}) \\
 &= \overline{\overline{A}(A + B) + (A \oplus B)[(A \oplus B) + b_i]} \\
 &= \overline{A + (A + B) + (A \oplus B) + (A \oplus B) + b_i}
 \end{aligned}$$

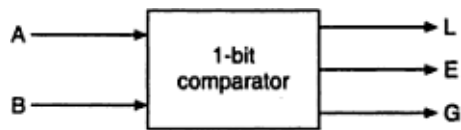




Logic diagram of a full subtractor using only 2-input NOR gates.

Comparators:

$$\text{EQUALITY} = (A_3 \odot B_3)(A_2 \odot B_2)(A_1 \odot B_1)(A_0 \odot B_0)$$



Block diagram of a 1-bit comparator.



The logic for a 1-bit magnitude comparator: Let the 1-bit numbers be $A = A_0$ and $B = B_0$.

If $A_0 = 1$ and $B_0 = 0$, then $A > B$.

Therefore,

$$A > B : G = A_0 \bar{B}_0$$

If $A_0 = 0$ and $B_0 = 1$, then $A < B$.

Therefore,

$$A < B : L = \bar{A}_0 B_0$$

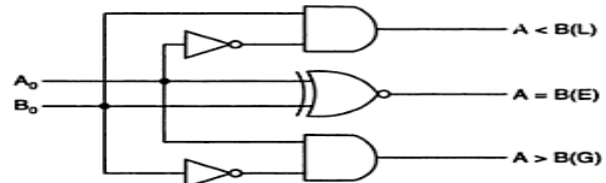
If A_0 and B_0 coincide, i.e. $A_0 = B_0 = 0$ or if $A_0 = B_0 = 1$, then $A = B$.

Therefore,

$$A = B : E = A_0 \odot B_0$$

A_0	B_0	L	E	G
0	0	0	1	0
0	1	1	0	0
1	0	0	0	1
1	1	0	1	0

(a) Truth table



(b) Logic diagram
1-bit comparator.

1. Magnitude Comparator:

1-bit Magnitude Comparator:

The logic for a 2-bit magnitude comparator: Let the two 2-bit numbers be $A = A_1 A_0$ and $B = B_1 B_0$.

1. If $A_1 = 1$ and $B_1 = 0$, then $A > B$ or

2. If A_1 and B_1 coincide and $A_0 = 1$ and $B_0 = 0$, then $A > B$. So the logic expression for $A > B$ is

$$A > B : G = A_1 \bar{B}_1 + (A_1 \odot B_1) A_0 \bar{B}_0$$

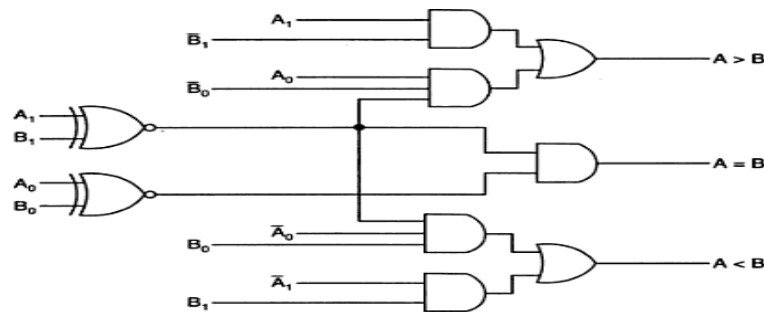
1. If $A_1 = 0$ and $B_1 = 1$, then $A < B$ or

2. If A_1 and B_1 coincide and $A_0 = 0$ and $B_0 = 1$, then $A < B$. So the expression for $A < B$ is

$$A < B : L = \bar{A}_1 B_1 + (A_1 \odot B_1) \bar{A}_0 B_0$$

If A_1 and B_1 coincide and if A_0 and B_0 coincide then $A = B$. So the expression for $A = B$ is

$$A = B : E = (A_1 \odot B_1)(A_0 \odot B_0)$$



Logic diagram of a 2-bit magnitude comparator.

4- Bit Magnitude Comparator:

The logic for a 4-bit magnitude comparator: Let the two 4-bit numbers be $A = A_3A_2A_1A_0$ and $B = B_3B_2B_1B_0$.

1. If $A_3 = 1$ and $B_3 = 0$, then $A > B$. Or
2. If A_3 and B_3 coincide, and if $A_2 = 1$ and $B_2 = 0$, then $A > B$. Or
3. If A_3 and B_3 coincide, and if A_2 and B_2 coincide, and if $A_1 = 1$ and $B_1 = 0$, then $A > B$. Or
4. If A_3 and B_3 coincide, and if A_2 and B_2 coincide, and if A_1 and B_1 coincide, and if $A_0 = 1$ and $B_0 = 0$, then $A > B$.

From these statements, we see that the logic expression for $A > B$ can be written as

$$(A > B) = A_3\bar{B}_3 + (A_3 \odot B_3)A_2\bar{B}_2 + (A_3 \odot B_3)(A_2 \odot B_2)A_1\bar{B}_1 + (A_3 \odot B_3)(A_2 \odot B_2)(A_1 \odot B_1)A_0\bar{B}_0$$

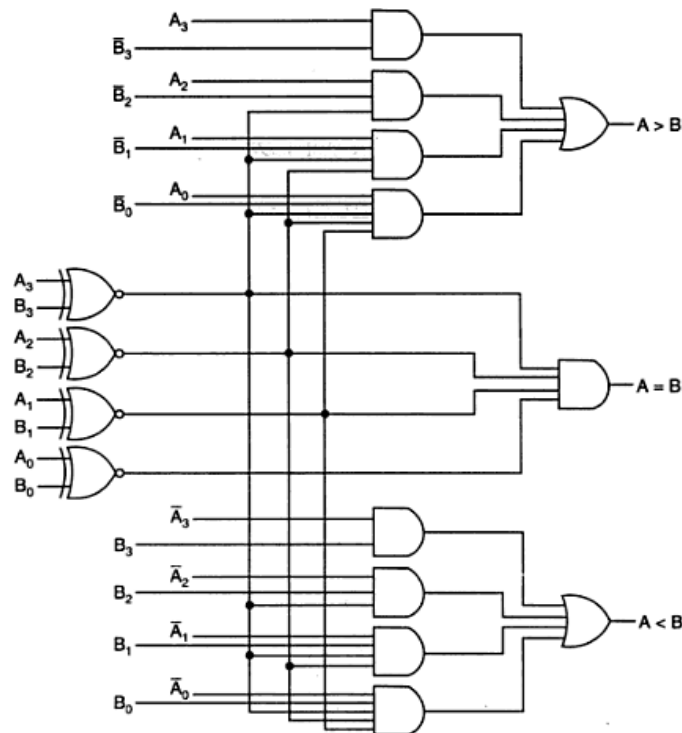
Similarly, the logic expression for $A < B$ can be written as

$$A < B = \bar{A}_3B_3 + (A_3 \odot B_3)\bar{A}_2B_2 + (A_3 \odot B_3)(A_2 \odot B_2)\bar{A}_1B_1 + (A_3 \odot B_3)(A_2 \odot B_2)(A_1 \odot B_1)\bar{A}_0B_0$$

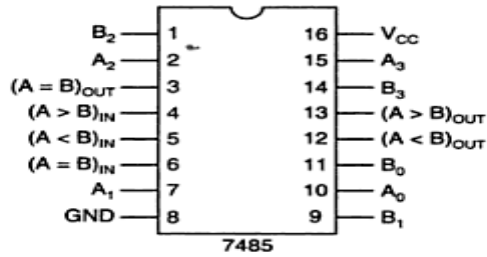
If A_3 and B_3 coincide and if A_2 and B_2 coincide and if A_1 and B_1 coincide and if A_0 and B_0 coincide, then $A = B$.

So the expression for $A = B$ can be written as

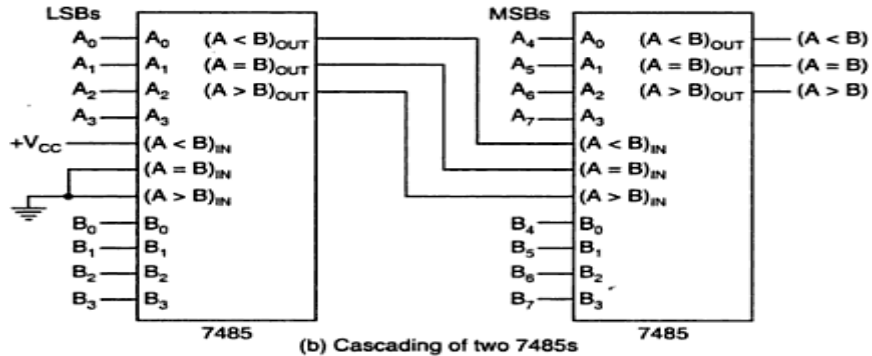
$$(A = B) = (A_3 \odot B_3)(A_2 \odot B_2)(A_1 \odot B_1)(A_0 \odot B_0)$$



IC Comparator:



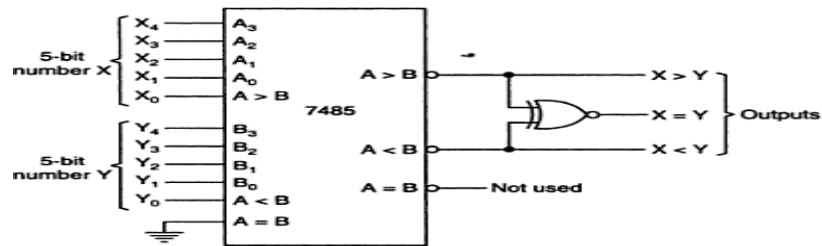
(a) Pin diagram of 7485



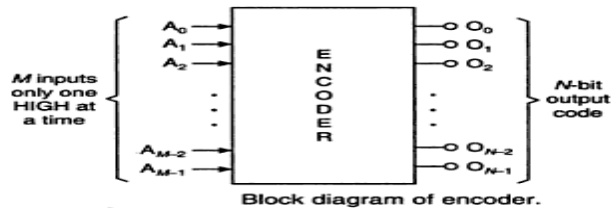
(b) Cascading of two 7485s

Pin diagram and cascading of 7485 4-bit comparators.

ENCODERS:



Use of 7485 as a 5-bit comparator.

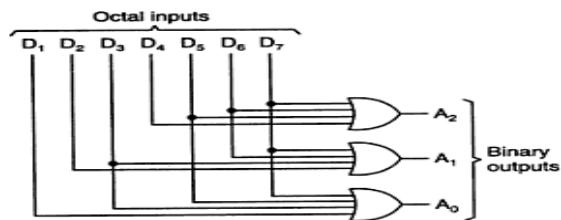


Block diagram of encoder.

Octal to Binary Encoder:

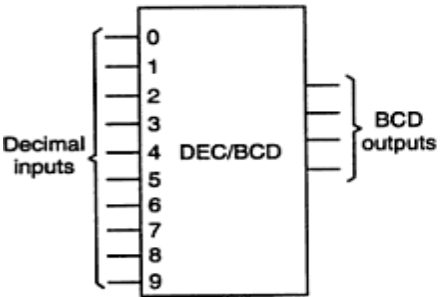
Octal digits	Binary		
	A ₂	A ₁	A ₀
D ₀	0	0	0
D ₁	0	0	1
D ₂	0	1	0
D ₃	0	1	1
D ₄	1	0	0
D ₅	1	0	1
D ₆	1	1	0
D ₇	1	1	1

(a) Truth table



(b) Logic diagram
Octal-to-binary encoder.

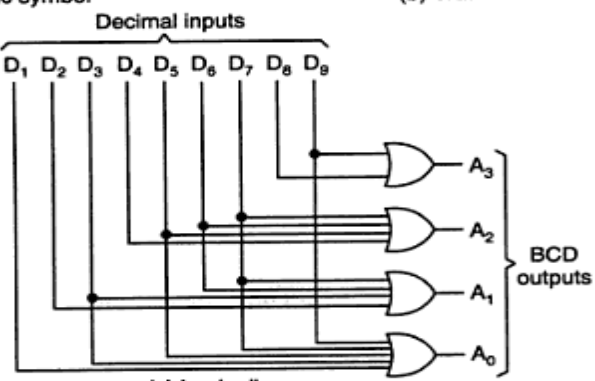
Decimal to BCD Encoder:



(a) Logic symbol

Decimal inputs	Binary			
	A ₃	A ₂	A ₁	A ₀
D ₀	0	0	0	0
D ₁	0	0	0	1
D ₂	0	0	1	0
D ₃	0	0	1	1
D ₄	0	1	0	0
D ₅	0	1	0	1
D ₆	0	1	1	0
D ₇	0	1	1	1
D ₈	1	0	0	0
D ₉	1	0	0	1

(b) Truth table



(c) Logic diagram

Decimal-to-BCD encoder.

UNIT-III

SEQUENTIAL CIRCUITS

The Basic Latch

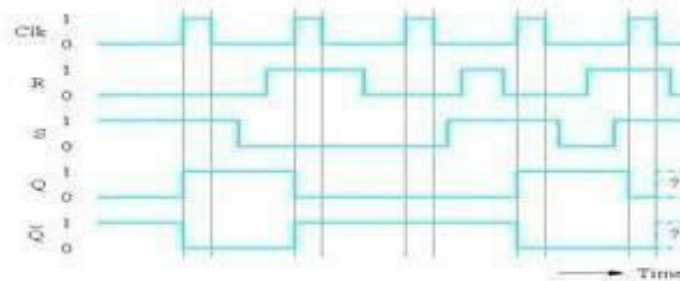
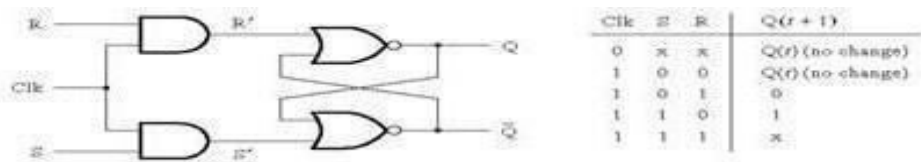
- **Basic latch** is a feedback connection of two NOR gates or two NAND gates
- It can store one bit of information

It can be set to 1 using the S input and reset to 0 using the R input

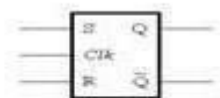
The Gated Latch

- **Gated latch** is a basic latch that includes input gating and a control signal
- The latch retains its existing state when the control input is equal to 0
- Its state may be changed when the control signal is equal to 1. In our discussion we referred to the control input as the clock
- We consider two types of gated latches:
 - **Gated SR latch** uses the S and R inputs to set the latch to 1 or reset it to 0, respectively.
 - **Gated D latch** uses the D input to force the latch into a state that has the same logic value as the D input.

Gated S/R Latch

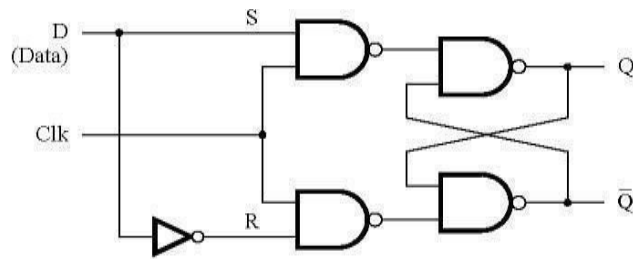


(c) Timing diagram



(d) Graphical symbol

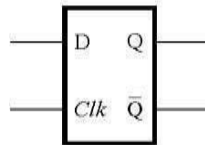
Gated D Latch



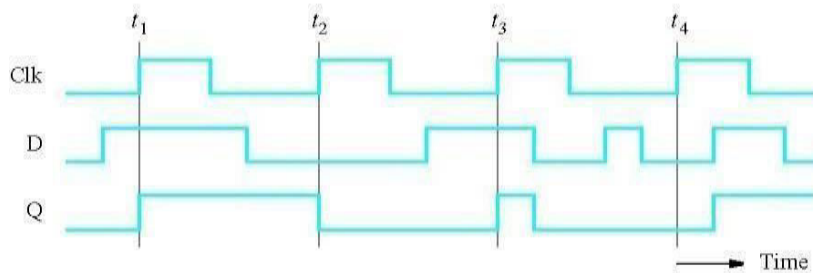
(a) Circuit

Clk	D	$Q(t+1)$
0	x	$Q(t)$
1	0	0
1	1	1

(b) Characteristic table



(c) Graphical symbol



(d) Timing diagram

Setup and Hold Times

● Setup Time t_{su}

The minimum time that the input signal must be stable prior to the edge of the clock signal.

● Hold Time t_h

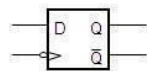
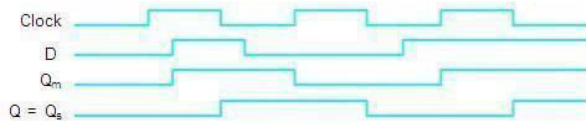
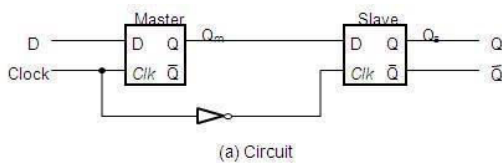
The minimum time that the input signal must be stable after the edge of the clock signal.

Flip-Flops

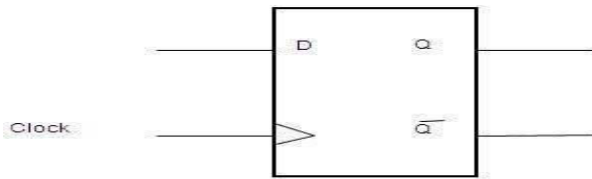
- A **flip-flop** is a storage element based on the gated latch principle
- It can have its output state changed only on the edge of the controlling clocksignal
- We consider two types:

- **Edge-triggered flip-flop** is affected only by the input values present when the active edge of the clock occurs
- **Master-slave flip-flop** is built with two gated latches
- The master stage is active during half of the clock cycle, and the slave stage is active during the other half.
- The output value of the flip-flop changes on the edge of the clock that activates the transfer into the slave stage.

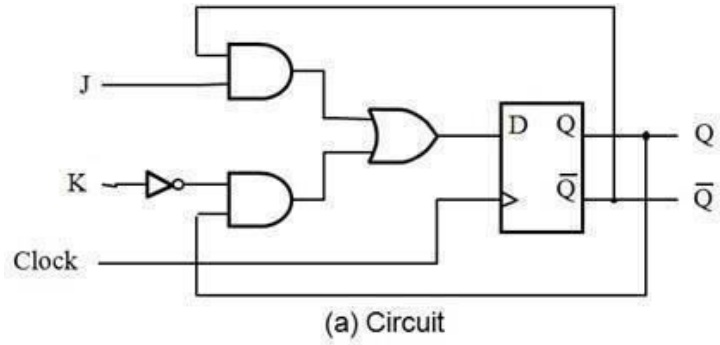
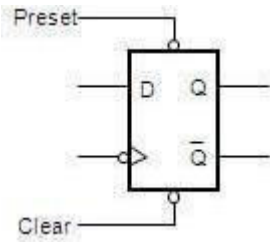
Master-Slave D Flip-Flop



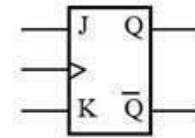
A Positive-Edge-Triggered D Flip-Flop



Master-Slave D Flip-Flop with *Clear* and *Preset*

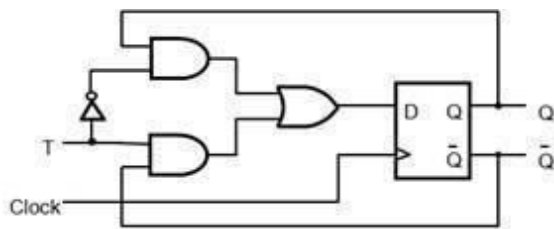


J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\bar{Q}(t)$



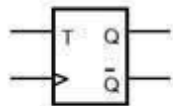
(b) Characteristic table (c) Graphical symbol

T Flip-Flop

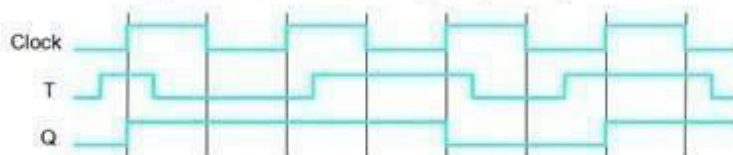


(a) Circuit

T	Q(t+1)
0	Q(t)
1	$\bar{Q}(t)$



(b) Characteristic table (c) Graphical symbol



(d) Timing diagram

Excitation Tables

Previous State -> Present State	D
0 -> 0	0
0 -> 1	1
1 -> 0	0
1 -> 1	1

Previous State -> Present State	J	K
0 -> 0	0	X
0 -> 1	1	X
1 -> 0	X	1
1 -> 1	X	0

Previous State -> Present State	S	R
0 -> 0	0	X
0 -> 1	1	0
1 -> 0	0	1
1 -> 1	X	0

Previous State -> Present State	T
0 -> 0	0
0 -> 1	1
1 -> 0	1
1 -> 1	0

Conversions of flip-flops

Example: Use JK-FF to realize D-FF

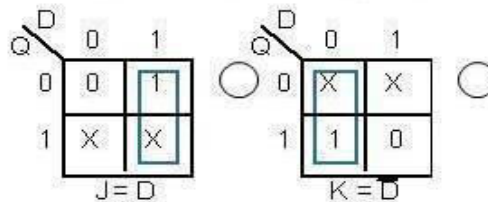
- 1) Start transition table for D-FF
- 2) Create K-maps to express J and K as functions of inputs (D, Q)
- 3) Fill in K-maps with appropriate values for J and K to cause the same state transition as in the D-FF transition table

D	Q	Q ⁺	J	K
0	0	0	0	X
0	1	0	X	1
1	0	1	1	X
1	1	1	X	0

State-Table

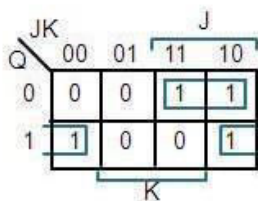
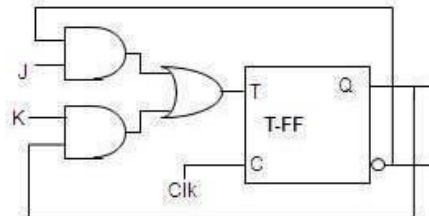
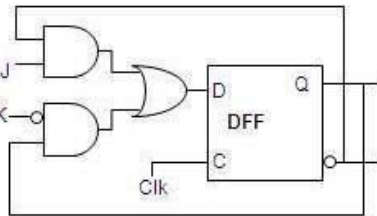
e.g.
when D=Q=0, then Q⁺=0
the same transition Q⁻→Q⁺
is realized with J=0, K=X

Q	Q ⁺	R	S	J	K	T	D
0	0	X	0	0	X	0	0
0	1	0	1	1	X	1	1
1	0	1	0	X	1	1	0
1	1	0	X	X	0	0	1

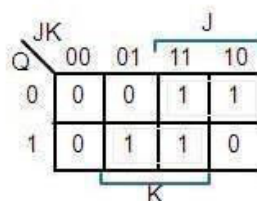


Example: Implement JK-FF using a D-FF

J	K	Q	Q ⁺	D	T
0	0	0	0	0	0
0	0	1	1	1	0
0	1	0	0	0	0
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1



$$d = jQ + Kq$$



$$t = jQ + kq$$

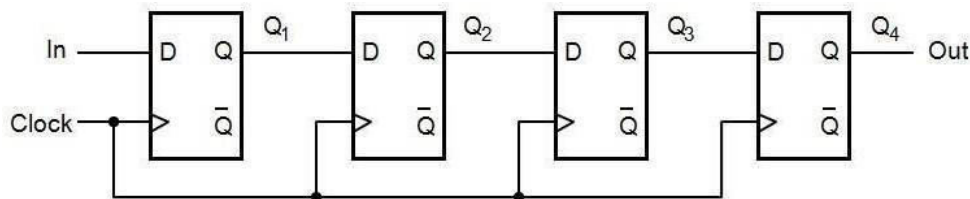
Sequential Circuit Design

- Steps in the design process for sequential circuits
- State Diagrams and State Tables □ Examples
- Steps in Design of a Sequential Circuit
 - 1. Specification – A description of the sequential circuit. Should include a detailing of the inputs, the outputs, and the operation. Possibly assumes that you have knowledge of digital system basics.
 - 2. Formulation: Generate a state diagram and/or a state table from the statement of the problem.
 - 3. State Assignment: From a state table assign binary codes to the states.
 - 4. Flip-flop Input Equation Generation: Select the type of flip-flop for the circuit and generate the needed input for the required state transitions
 - 5. Output Equation Generation: Derive output logic equations for generation of the output from the inputs and current state.
 - 6. Optimization: Optimize the input and output equations. Today, CAD systems are typically used for this in real systems.
 - 7. Technology Mapping: Generate a logic diagram of the circuit using ANDs, ORs, Inverters, and F/Fs.
 - 8. Verification: Use a HDL to verify the design

Registers and Counters

- An n -bit register is a cascade of n flip-flops and can store an n -bit binary data
- A counter can count occurrences of events and can generate timing intervals for control purposes

A Simple Shift Register

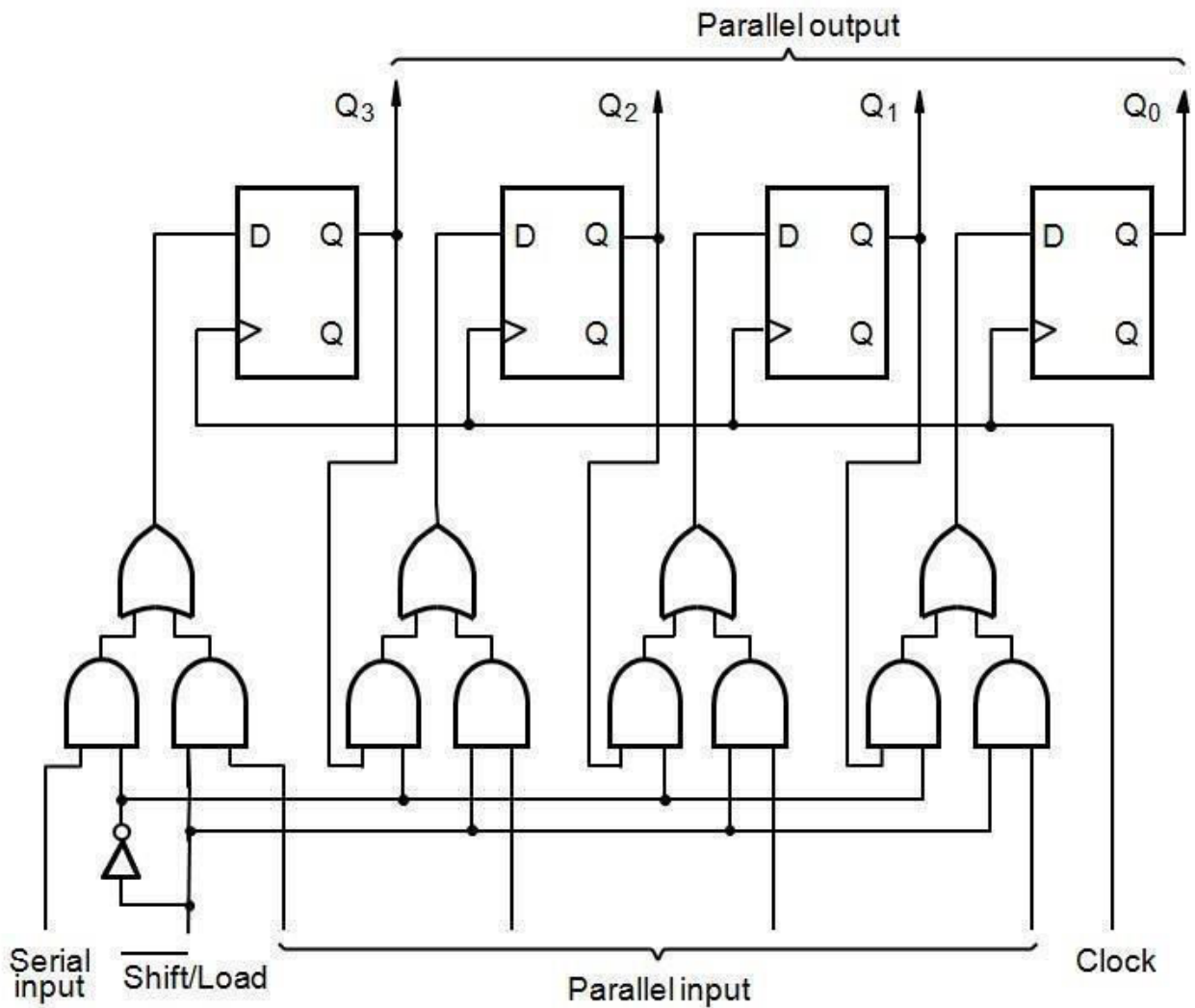


(a) Circuit

	In	Q ₁	Q ₂	Q ₃	Q ₄ = Out
t_0	1	0	0	0	0
t_1	0	1	0	0	0
t_2	1	0	1	0	0
t_3	1	1	0	1	0
t_4	1	1	1	0	1
t_5	0	1	1	1	0
t_6	0	0	1	1	1
t_7	0	0	0	1	1

(b) A sample sequence

Parallel-Access Shift Register



Counters

- Counters are a specific type of sequential circuit.
- Like registers, the state, or the flip-flop values themselves, serves as the “output.”
- The output value increases by one on each clock cycle.
- After the largest value, the output “wraps around” back to 0.
- Using two bits, we’d get something like this:

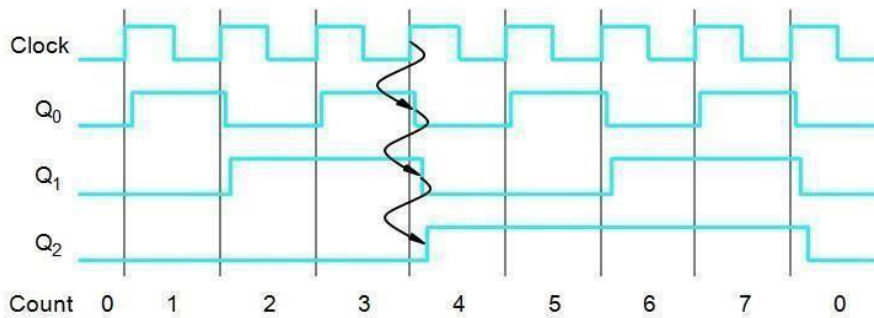
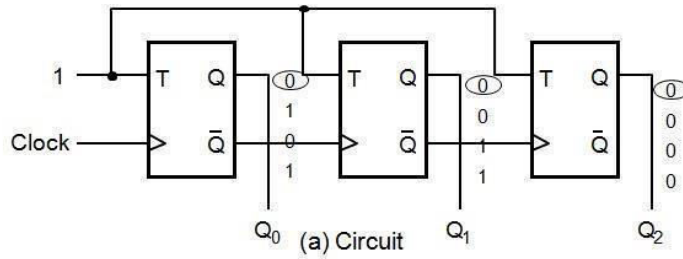
Present State		Next State	
A	B	A	B
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

Benefits of counters

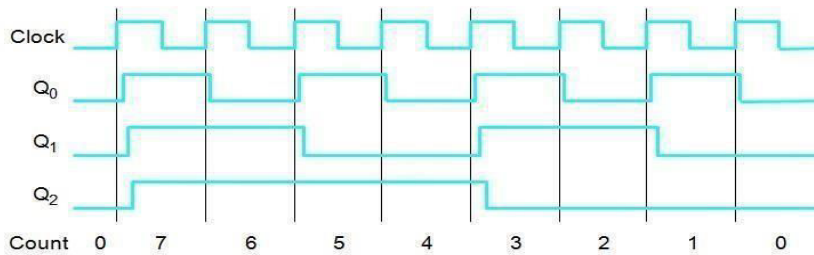
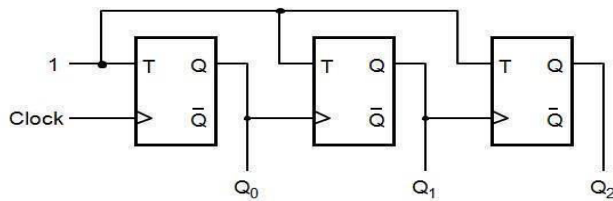
- Counters can act as simple clocks to keep track of “time.” •
 - You may need to record how many times something has happened.
 - How many bits have been sent or received?
 - How many steps have been performed in some computation?
- All processors contain a program counter, or PC.
 - Programs consist of a list of instructions that are to be executed one after another (for the most part).
 - The PC keeps track of the instruction currently being executed.
 - The PC increments once on each clock cycle, and the next program instruction is then executed.

A Three-Bit Up-Counter

- Q_1 is connected to clk, Q_2 and Q_3 are clocked by Q' of the preceding stage (hence called asynchronous or ripple counter)



A Three-Bit Down-Counter



Shift registers:

In digital circuits, a **shift register** is a cascade of flip-flops sharing the same clock, in which the output of each flip-flop is connected to the "data" input of the next flip-flop in the chain, resulting in a circuit that shifts by one position the "bit array" stored in it, *shifting in* the data present at its input and *shifting out* the last bit in the array, at each transition of the clock input. More generally, a **shift register** may be multidimensional, such that its "data in" and stage outputs are themselves bit arrays: this is implemented simply by running several shift registers of the same bit-length in parallel.

Shift registers can have both parallel and serial inputs and outputs. These are often configured as **serial-in, parallel-out (SIPO)** or as **parallel-in, serial-out (PISO)**. There are also types that have both serial and parallel input and types with serial and parallel output. There are also **bi-directional** shift registers which allow shifting in both directions: L→R or R→L. The serial input and last output of a shift register can also be connected to create a **circular shift register**

Shift registers are a type of logic circuits closely related to counters. They are basically for the storage and transfer of digital data.

Buffer register:

The buffer register is the simple set of registers. It simply stores the binary word. The buffer may be controlled buffer. Most of the buffer registers used D Flip-flops.

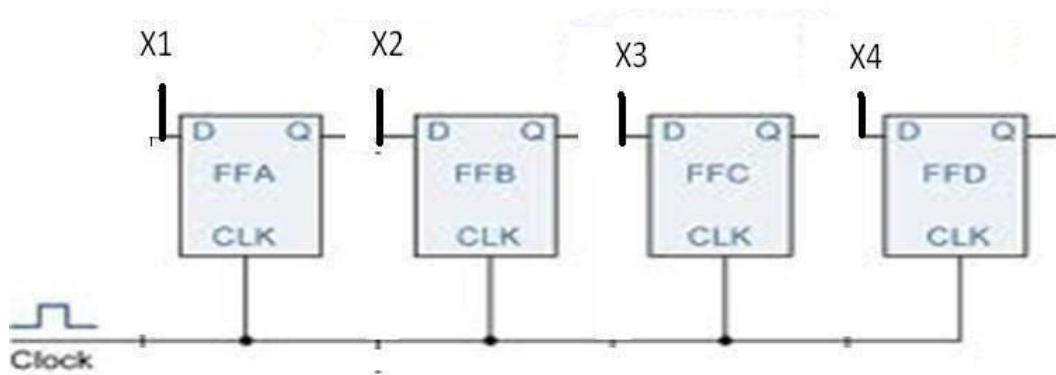


Figure: logic diagram of 4-bit buffer register

The figure shows a 4-bit buffer register. The binary word to be stored is applied to the data terminals. On the application of clock pulse, the output word becomes the same as the word applied at the terminals. i.e., the input word is loaded into the register by the application of clock pulse.

When the positive clock edge arrives, the stored word becomes:

$$Q_4Q_3Q_2Q_1 = X_4X_3X_2X_1$$
$$Q = X$$

Controlled buffer register:

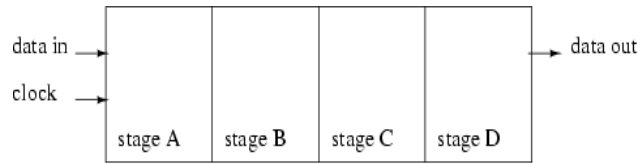
If goes LOW, all the FFs are RESET and the output becomes, $Q = 0000$.

When is HIGH, the register is ready for action. LOAD is the control input. When LOAD is HIGH, the data bits X can reach the D inputs of FF's.

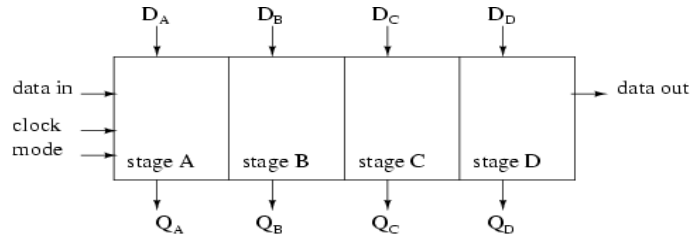
$$Q_4Q_3Q_2Q_1 = X_4X_3X_2X_1$$
$$Q = X$$

When load is low, the X bits cannot reach the FF's.

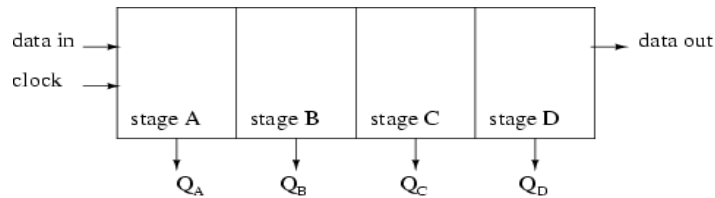
Data transmission in shift registers:



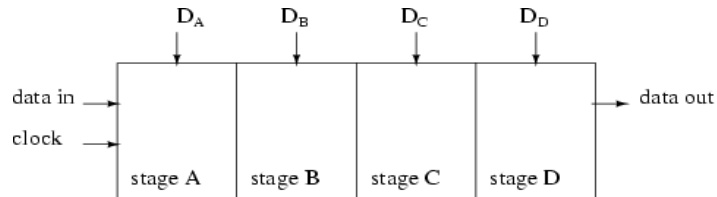
Serial-in, serial-out shift register with 4-stages



Parallel-in, parallel-out shift register with 4-stages



Serial-in, parallel-out shift register with 4-stages



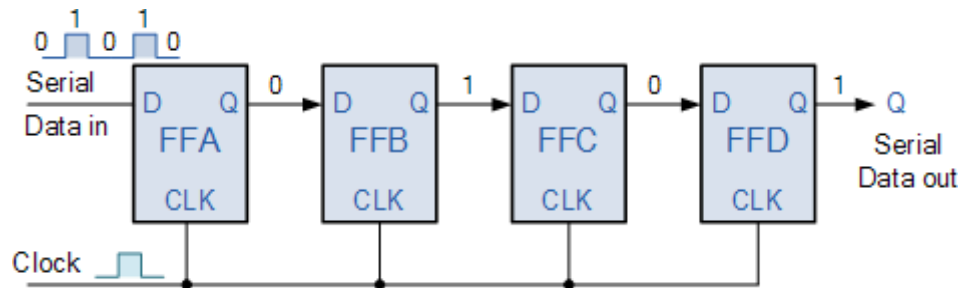
Parallel-in, serial-out shift register with 4-stages

A number of ff's connected together such that data may be shifted into and shifted out of them is called shift register. data may be shifted into or out of the register in serial form or in parallel form. There are four basic types of shift registers.

1. Serial in, serial out, shift right, shift registers
2. Serial in, serial out, shift left, shift registers
3. Parallel in, serial out shift registers
4. Parallel in, parallel out shift registers

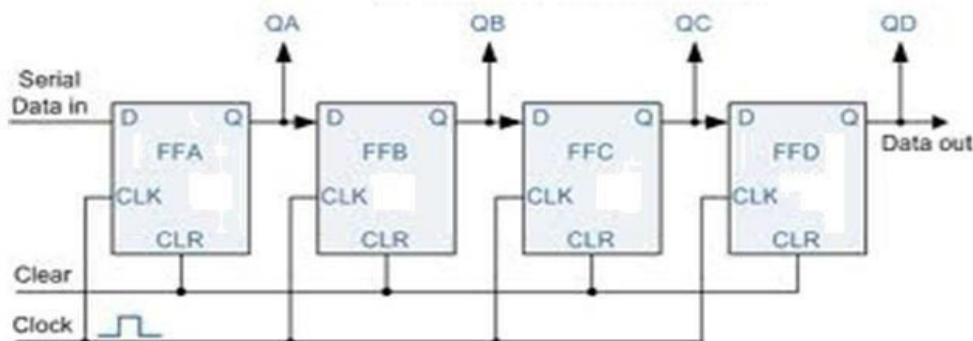
Serial IN, serial OUT, shift right, shift left register:

The logic diagram of 4-bit serial in serial out, right shift register with four stages. The register can store four bits of data. Serial data is applied at the input D of the first FF. the Q output of the first FF is connected to the D input of another FF. the data is outputted from the Q terminal of the last FF.



When serial data is transferred into a register, each new bit is clocked into the first FF at the positive going edge of each clock pulse. The bit that was previously stored by the first FF is transferred to the second FF. the bit that was stored by the Second FF is transferred to the third FF.

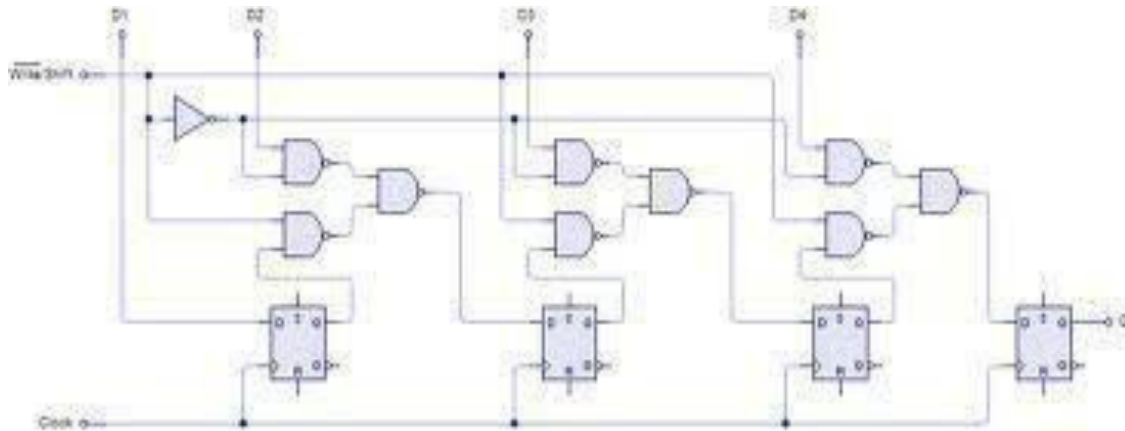
Serial-in, parallel-out, shift register:



In this type of register, the data bits are entered into the register serially, but the data stored in the register is shifted out in parallel form.

Once the data bits are stored, each bit appears on its respective output line and all bits are available simultaneously, rather than on a bit-by-bit basis with the serial output. The serial-in, parallel out, shift register can be used as serial-in, serial out, shift register if the output is taken from the Q terminal of the last FF.

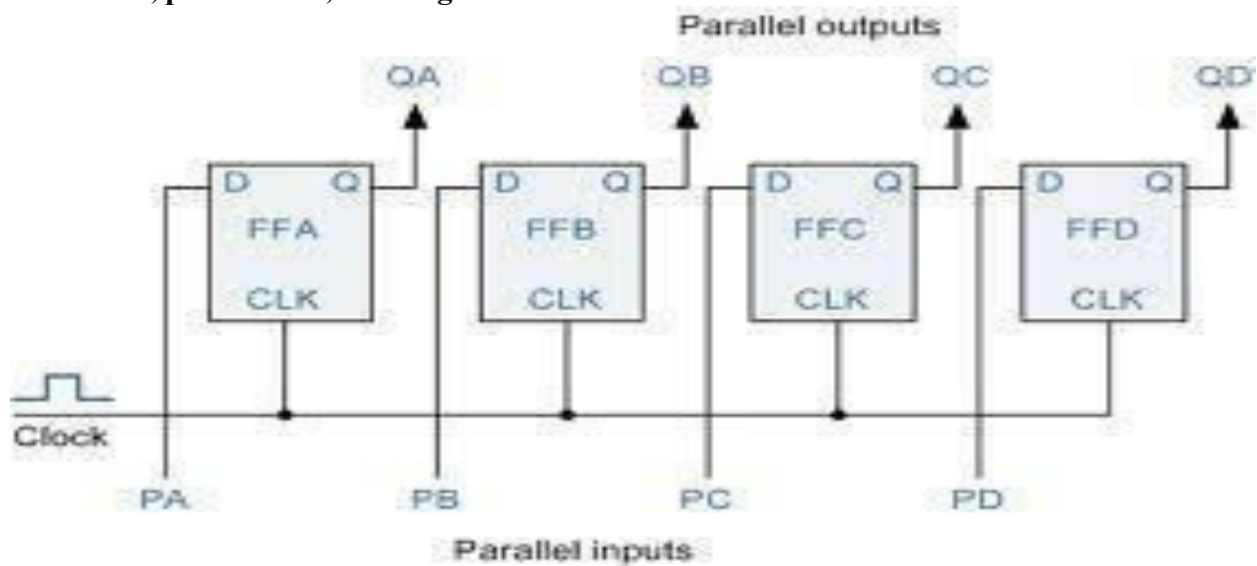
Parallel-in, serial-out, shift register:



For a parallel-in, serial out, shift register, the data bits are entered simultaneously into their respective stages on parallel lines, rather than on a bit-by-bit basis on one line as with serial data bits are transferred out of the register serially. On a bit-by-bit basis over a single line.

There are four data lines A,B,C,D through which the data is entered into the register in parallel form. The signal shift/ load allows the data to be entered in parallel form into the register and the data is shifted out serially from terminal Q4

Parallel-in, parallel-out, shift register



In a parallel-in, parallel-out shift register, the data is entered into the register in parallel form, and also the data is taken out of the register in parallel form. Data is applied to the D input terminals of the FF's. When a clock pulse is applied, at the positive going edge of the pulse, the D inputs are shifted into the Q outputs of the FFs. The register now stores the data. The stored data is available instantaneously for shifting out in parallel form.

Bidirectional shift register:

A bidirectional shift register is one which the data bits can be shifted from left to right or from right to left. A fig shows the logic diagram of a 4-bit serial-in, serial out, bidirectional shift register. Right/left is the mode signal, when right /left is a 1, the logic circuit works as a shift-register.the bidirectional operation is achieved by using the mode signal and two NAND gates and one OR gate for each stage.

A HIGH on the right/left control input enables the AND gates G1, G2, G3 and G4 and disables the AND gates G5,G6,G7 and G8, and the state of Q output of each FF is passed through the gate to the D input of the following FF. when a clock pulse occurs, the data bits are then effectively shifted one place to the right. A LOW on the right/left control inputs enables the AND gates G5, G6, G7 and G8 and disables the And gates G1, G2, G3 and G4 and the Q output of each FF is passed to the D input of the preceding FF. when a clock pulse occurs, the data bits are then effectively shifted one place to the left. Hence, the circuit works as a bidirectional shift register

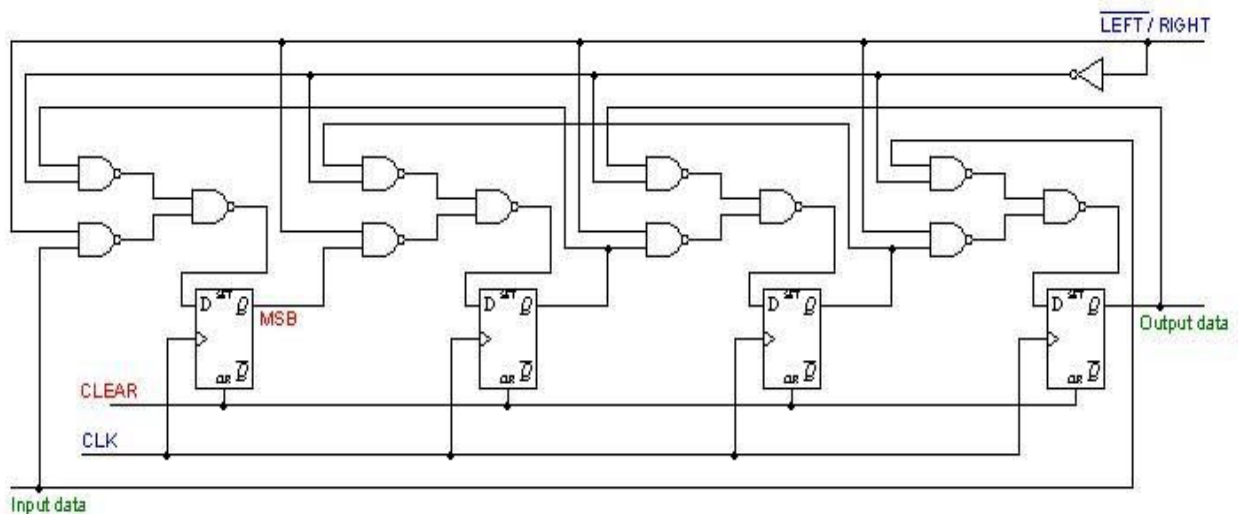


Figure: logic diagram of a 4-bit bidirectional shift register

Universal shift register:

A register is capable of shifting in one direction only is a unidirectional shift register. One that can shift both directions is a bidirectional shift register. If the register has both shifts and parallel load capabilities, it is referred to as a universal shift registers. Universal shift register is a bidirectional register, whose input can be either in serial form or in parallel form and whose output also can be in serial form or I parallel form.

The most general shift register has the following capabilities.

1. A clear control to clear the register to 0
2. A clock input to synchronize the operations
3. A shift-right control to enable the shift-right operation and serial input and output lines associated with the shift-right

4. A shift-left control to enable the shift-left operation and serial input and output lines associated with the shift-left
5. A parallel loads control to enable a parallel transfer and the n input lines associated with the parallel transfer
6. N parallel output lines
7. A control state that leaves the information in the register unchanged in the presence of the clock.

A universal shift register can be realized using multiplexers. The below fig shows the logic diagram of a 4-bit universal shift register that has all capabilities. It consists of 4 D flip-flops and four multiplexers. The four multiplexers have two common selection inputs s_1 and s_0 . Input 0 in each multiplexer is selected when $S_1S_0=00$, input 1 is selected when $S_1S_0=01$ and input 2 is selected when $S_1S_0=10$ and input 4 is selected when $S_1S_0=11$. The selection inputs control the mode of operation of the register according to the functions entries. When $S_1S_0=0$, the present value of the register is applied to the D inputs of flip-flops. The condition forms a path from the output of each flip-flop into the input of the same flip-flop. The next clock edge transfers into each flip-flop the binary value it held previously, and no change of state occurs. When $S_1S_0=01$, terminal 1 of the multiplexer inputs have a path to the D inputs of the flip-flop. This causes a shift-right operation, with serial input transferred into flip-flop A_4 . When $S_1S_0=10$, a shift left operation results with the other serial input going into flip-flop A_1 . Finally when $S_1S_0=11$, the binary information on the parallel input lines is transferred into the register simultaneously during the next clock cycle

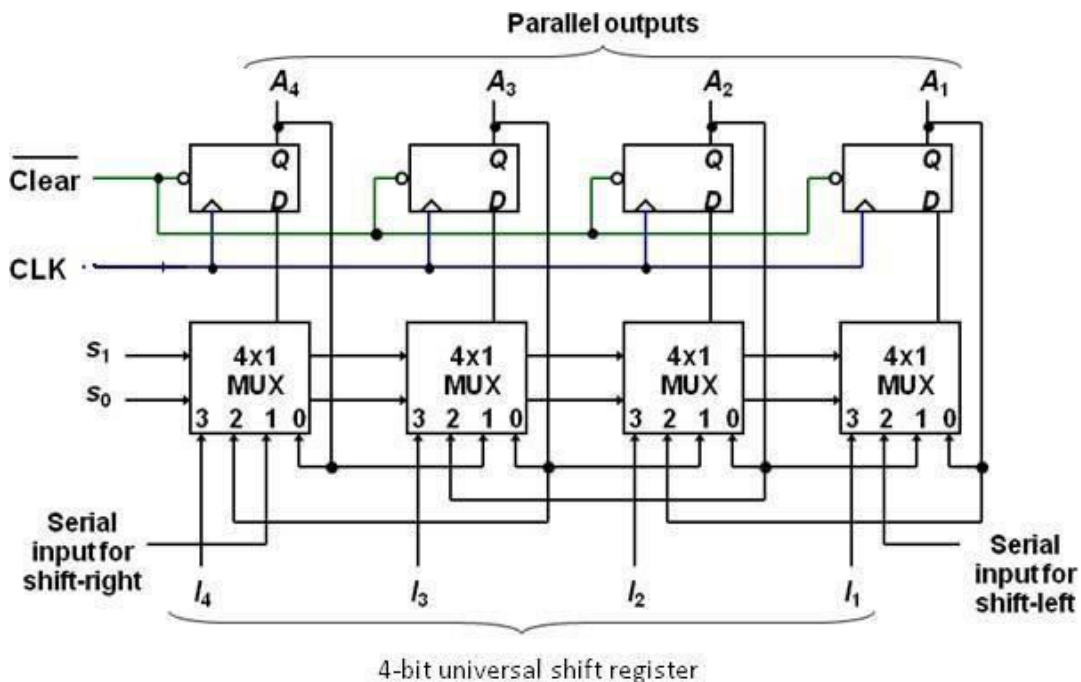


Figure: logic diagram 4-bit universal shift register

Function table for the register

mode control		
S0	S1	register operation
0	0	No change
0	1	Shift Right
1	0	Shift left
1	1	Parallel load

Counters:

Counter is a device which stores (and sometimes displays) the number of times particular event or process has occurred, often in relationship to a clock signal. A Digital counter is a set of flip flops whose state change in response to pulses applied at the input to the counter. Counters may be asynchronous counters or synchronous counters. Asynchronous counters are also called ripple counters

In electronics counters can be implemented quite easily using register-type circuits such as the flip-flops and a wide variety of classifications exist:

- Asynchronous (ripple) counter – changing state bits are used as clocks to subsequent state flip-flops
- Synchronous counter – all state bits change under control of a single clock
- Decade counter – counts through ten states per stage
- Up/down counter – counts both up and down, under command of a control input
- Ring counter – formed by a shift register with feedback connection in a ring
- Johnson counter – a *twisted* ring counter
- Cascaded counter
- Modulus counter.

Each is useful for different applications. Usually, counter circuits are digital in nature, and count in natural binary. Many types of counter circuits are available as digital building blocks, for example a number of chips in the 4000 series implement different counters.

Occasionally there are advantages to using a counting sequence other than the natural binary sequence such as the binary coded decimal counter, a linear feed-back shift register counter, or a gray-code counter.

Counters are useful for digital clocks and timers, and in oven timers, VCR clocks, etc.

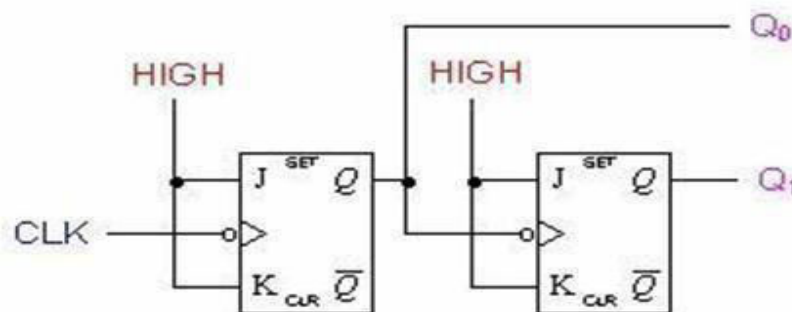
Asynchronous counters:

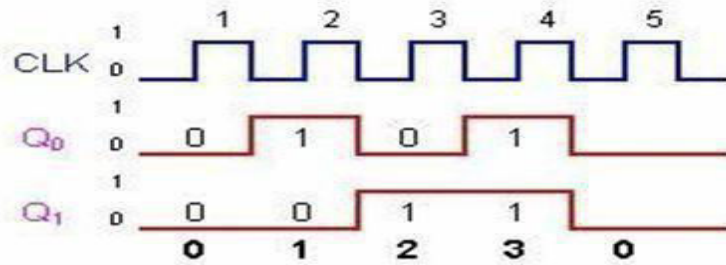
An asynchronous (ripple) counter is a single [JK-type flip-flop](#), with its J (data) input fed from its own inverted output. This circuit can store one bit, and hence can count from zero to one before it overflows (starts over from 0). This counter will increment once for every clock cycle and takes two clock cycles to overflow, so every cycle it will alternate between a transition from 0 to 1 and a transition from 1 to 0. Notice that this creates a new clock with a 50% [duty cycle](#) at exactly half the frequency of the input clock. If this output is then used as the clock signal for a similarly arranged D flip-flop (remembering to invert the output to the input), one will get another 1 bit counter that counts half as fast. Putting them together yields a two-bit counter:

Two-bit ripple up-counter using negative edge triggered flip flop:

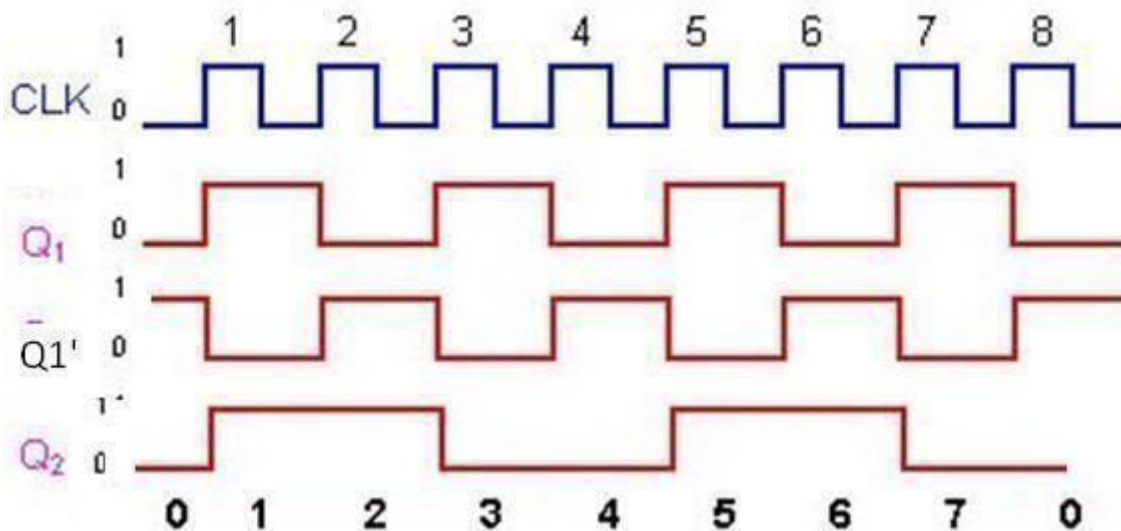
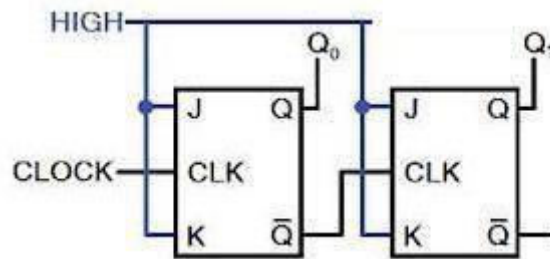
Two bit ripple counter used two flip-flops. There are four possible states from 2 – bit up-counting I.e. 00, 01, 10 and 11.

- The counter is initially assumed to be at a state 00 where the outputs of the two flip-flops are noted as Q_1Q_0 . Where Q_1 forms the MSB and Q_0 forms the LSB.
- For the negative edge of the first clock pulse, output of the first flip-flop FF_1 toggles its state. Thus Q_1 remains at 0 and Q_0 toggles to 1 and the counter state are now read as 01.
- During the next negative edge of the input clock pulse FF_1 toggles and $Q_0 = 0$. The output Q_0 being a clock signal for the second flip-flop FF_2 and the present transition acts as a negative edge for FF_2 thus toggles its state $Q_1 = 1$. The counter state is now read as 10.
- For the next negative edge of the input clock to FF_1 output Q_0 toggles to 1. But this transition from 0 to 1 being a positive edge for FF_2 output Q_1 remains at 1. The counter state is now read as 11.
- For the next negative edge of the input clock, Q_0 toggles to 0. This transition from 1 to 0 acts as a negative edge clock for FF_2 and its output Q_1 toggles to 0. Thus the starting state 00 is attained. Figure shown below





Two-bit ripple down-counter using negative edge triggered flip flop:



A 2-bit down-counter counts in the order 0,3,2,1,0,1.....,i.e, 00,11,10,01,00,11,etc. the above fig. shows ripple down counter, using negative edge triggered J-K FFs and its timing diagram.

- For down counting, $Q1'$ of FF1 is connected to the clock of Ff2. Let initially all the FF1 toggles, so, $Q1$ goes from a 0 to a 1 and $Q1'$ goes from a 1 to a 0.

- The negative-going signal at Q_1' is applied to the clock input of FF2, toggles FF2 and, therefore, Q_2 goes from a 0 to a 1. so, after one clock pulse $Q_2=1$ and $Q_1=1$, I.e., the state of the counter is 11.
- At the negative-going edge of the second clock pulse, Q_1 changes from a 1 to a 0 and Q_1' from a 0 to a 1.
- This positive-going signal at Q_1' does not affect FF2 and, therefore, Q_2 remains at a 1. Hence, the state of the counter after second clock pulse is 10
- At the negative going edge of the third clock pulse, FF1 toggles. So Q_1 , goes from a 0 to a 1 and Q_1' from 1 to 0. This negative going signal at Q_1' toggles FF2 and, so, Q_2 changes from 1 to 0, hence, the state of the counter after the third clock pulse is 01.
- At the negative going edge of the fourth clock pulse, FF1 toggles. So Q_1 , goes from a 1 to a 0 and Q_1' from 0 to 1. . This positive going signal at Q_1' does not affect FF2 and, so, Q_2 remains at 0, hence, the state of the counter after the fourth clock pulse is 00.

Two-bit ripple up-down counter using negative edge triggered flip flop:

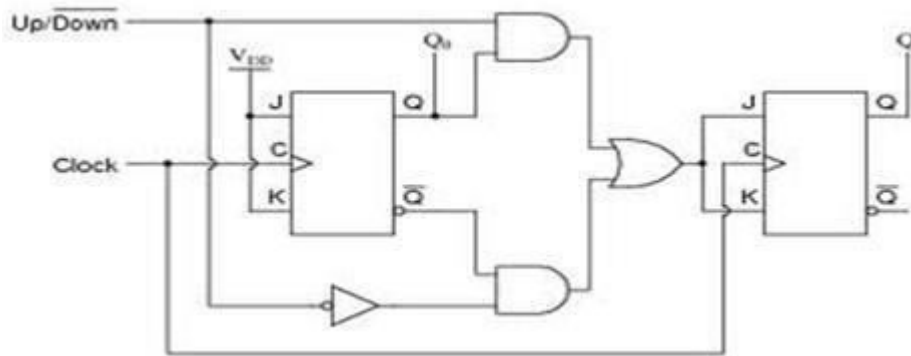


Figure: asynchronous 2-bit ripple up-down counter using negative edge triggered flip flop:

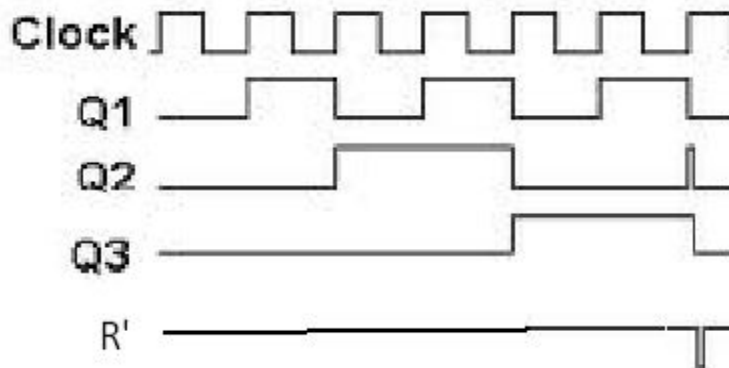
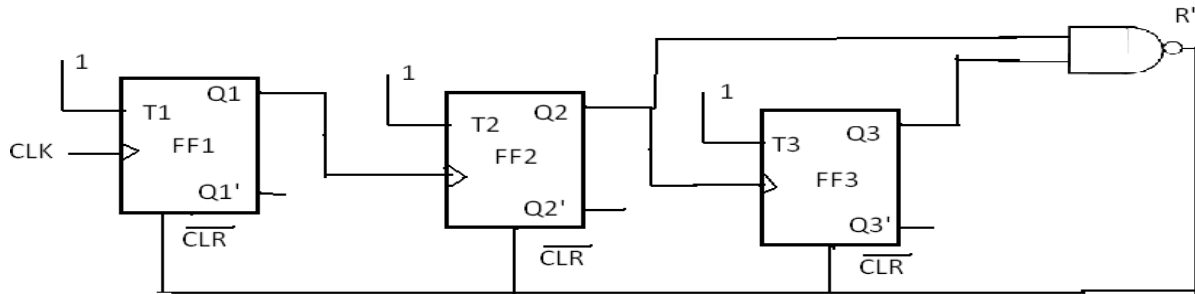
- As the name indicates an up-down counter is a counter which can count both in upward and downward directions. An up-down counter is also called a forward/backward counter or a bidirectional counter. So, a control signal or a mode signal M is required to choose the direction of count. When $M=1$ for up counting, Q_1 is transmitted to clock of FF2 and when $M=0$ for down counting, Q_1' is transmitted to clock of FF2. This is achieved by using two AND gates and one OR gates. The external clock signal is applied to FF1.
- Clock signal to FF2 = $(Q_1 \cdot \text{Up}) + (Q_1' \cdot \text{Down}) = Q_1 M + Q_1' M'$

Design of Asynchronous counters:

To design a asynchronous counter, first we write the sequence, then tabulate the values of reset signal R for various states of the counter and obtain the minimal expression for R and R' using K-Map or any other method. Provide a feedback such that R and R' resets all the FF's after the desired count

Design of a Mod-6 asynchronous counter using T FFs:

A mod-6 counter has six stable states 000, 001, 010, 011, 100, and 101. When the sixth clock pulse is applied, the counter temporarily goes to 110 state, but immediately resets to 000 because of the feedback provided. It is a divide-by-6 counter, in the sense that it divides the input clock frequency by 6. It requires three FFs, because the smallest value of n satisfying the condition $N \leq 2^n$ is $n=3$; three FFs can have 8 possible states, out of which only six are utilized and the remaining two states 110 and 111, are invalid. If initially the counter is in 000 state, then after the sixth clock pulse, it goes to 001, after the second clock pulse, it goes to 010, and so on.



After sixth clock pulse it goes to 000. For the design, write the truth table with present state outputs Q_3 , Q_2 and Q_1 as the variables, and reset R as the output and obtain an expression for R in terms of Q_3 , Q_2 , and Q_1 that decides the feedback into to be provided. From the truth table, $R = Q_3Q_2$. For active-low Reset, R' is used. The reset pulse is of very short duration, of the order of nanoseconds and it is equal to the propagation delay time of the NAND gate used. The expression for R can also be determined as follows.

$$R=0 \text{ for } 000 \text{ to } 101, R=1 \text{ for } 110, \text{ and } R=X \text{ for } 111$$

Therefore,

$$R = Q_3Q_2Q_1' + Q_3Q_2Q_1 = Q_3Q_2$$

The logic diagram and timing diagram of Mod-6 counter is shown in the above fig.

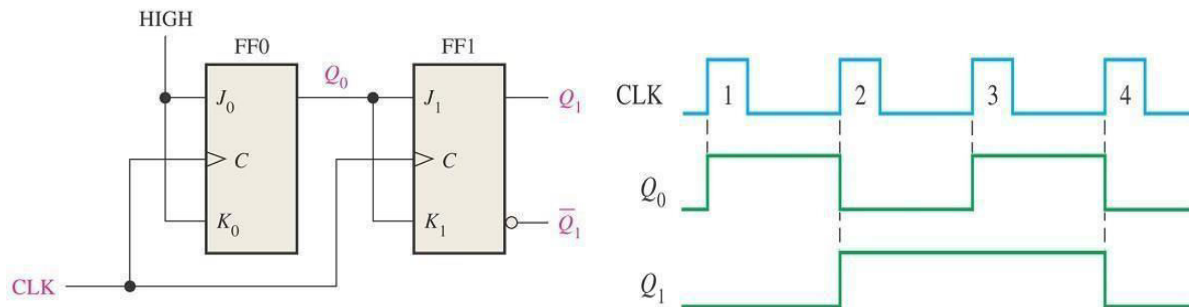
The truth table is as shown in below.

		Q2Q1			
		00	01	11	10
Q4Q3	00				
	01				
	11	X	X	X	X
	10		X	X	1

After pulses	Count			
	Q4	Q3	Q2	Q1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	0	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	0	1	0	1
10	0	0	0	0

Synchronous counters:

Asynchronous counters are serial counters. They are slow because each FF can change state only if all the preceding FFs have changed their state. If the clock frequency is very high, the asynchronous counter may skip some of the states. This problem is overcome in synchronous counters or parallel counters. Synchronous counters are counters in which all the flip flops are triggered simultaneously by the clock pulses. Synchronous counters have a common clock pulse applied simultaneously to all flip-flops. □ A 2-Bit Synchronous Binary Counter



Design of synchronous counters:

For a systematic design of synchronous counters. The following procedure is used.

Step 1: State Diagram: draw the state diagram showing all the possible states state diagram which also be called nth transition diagrams, is a graphical means of depicting the sequence of states through which the counter progresses.

Step 2: number of flip-flops: based on the description of the problem, determine the required number n of the flip-flops- the smallest value of n is such that the number of states $N \leq 2^n$ --- and the desired counting sequence.

Step 3: choice of flip-flops excitation table: select the type of flip-flop to be used and write the excitation table. An excitation table is a table that lists the present state (ps) , the next state(ns) and required excitations.

Step4: minimal expressions for excitations: obtain the minimal expressions for the excitations of the FF using K-maps drawn for the excitation of the flip-flops in terms of the present states and inputs.

Step5: logic diagram: draw a logic diagram based on the minimal expressions

Design of a synchronous 3-bit up-down counter using JK flip-flops:

Step1: determine the number of flip-flops required. A 3-bit counter requires three FFs. It has 8 states (000,001,010,011,101,110,111) and all the states are valid. Hence no don't cares. For selecting up and down modes, a control or mode signal M is required. When the mode signal M=1 and counts down when M=0. The clock signal is applied to all the FFs simultaneously.

Step2: draw the state diagrams: the state diagram of the 3-bit up-down counter is drawn as

Step3: select the type of flip flop and draw the excitation table: JK flip-flops are selected and the excitation table of a 3-bit up-down counter using JK flip-flops is drawn as shown in fig.

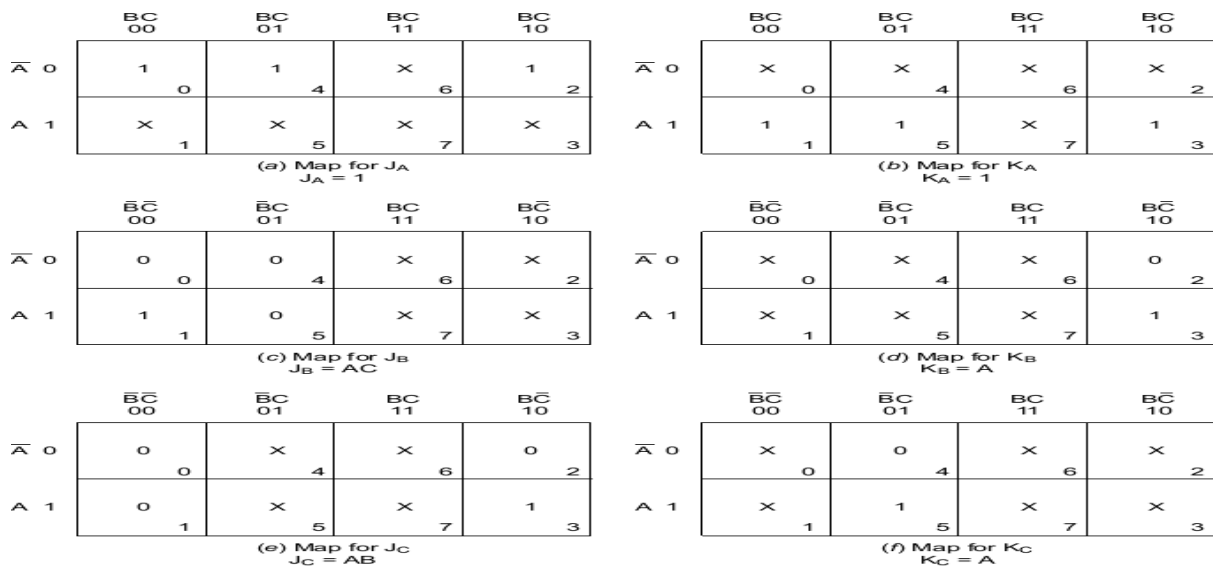
PS			mode	NS			required excitations					
Q3	Q2	Q1	M	Q3	Q2	Q1	J3	K3	J2	K2	J1	K1
0	0	0	0	1	1	1	1	x	1	x	1	x
0	0	0	1	0	0	1	0	x	0	x	1	x
0	0	1	0	0	0	0	0	x	0	x	x	1
0	0	1	1	0	1	0	0	x	1	x	x	1
0	1	0	0	0	0	1	0	x	x	1	1	x
0	1	0	1	0	1	1	0	x	x	0	1	x
0	1	1	0	0	1	0	0	x	x	0	x	1
0	1	1	1	1	0	0	1	x	x	1	x	1
1	0	0	0	0	1	1	x	1	1	x	1	x
1	0	0	1	1	0	1	x	0	0	x	1	x
1	0	1	0	1	0	0	x	0	0	x	x	1
1	0	1	1	1	1	0	x	0	1	x	x	1
1	1	0	0	1	0	1	x	0	x	1	1	x
1	1	0	1	1	1	1	x	0	x	0	1	x
1	1	1	0	1	1	0	x	0	x	0	x	1
1	1	1	1	0	0	0	x	1	x	1	x	1

Step4: obtain the minimal expressions: From the excitation table we can conclude that J1=1 and K1=1, because all the entries for J1 and K1 are either X or 1. The K-maps for J3, K3, J2 and K2 based on the excitation table and the minimal expression obtained from them are shown in fig.

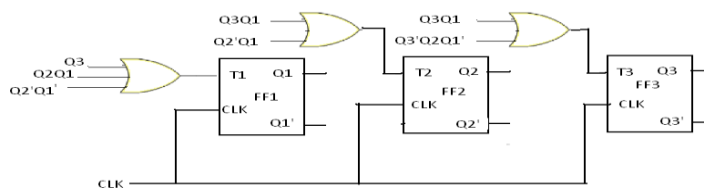
Step3: type of flip-flop and the excitation table: T flip-flops are selected and the excitation table of the mod-6 gray code counter using T-flip-flops is written as shown in fig.

PS			NS			required excitations		
Q3	Q2	Q1	Q3	Q2	Q1	T3	T2	T1
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	1	0
0	1	1	0	1	0	0	0	1
0	1	0	1	1	0	1	0	0
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1

Step4: The minimal expressions: the K-maps for excitations of FFs T3,T2,and T1 in terms of outputs of FFs Q3,Q2, and Q1, their minimization and the minimal expressions for excitations obtained from them are shown if fig



Step5: the logic diagram: the logic diagram based on those minimal expressions is drawn as shown in fig.



Design of a synchronous BCD Up-Down counter using FFs:

Step1: the number of flip-flops: a BCD counter is a mod-10 counter has 10 states (0000 through 1001) and so it requires $n=4$ FFs ($N \leq 2^n$, i.e., $10 \leq 2^4$). 4 FFs can have 16 states. So out of 16 states, six states (1010 through 1111) are invalid. For selecting up and down mode, a control or mode signal M is required. , it counts up when $M=1$ and counts down when $M=0$. The clock signal is applied to all FFs.

Step2: the state diagram: The state diagram of the mod-10 up-down counter is drawn as shown in fig.

Step3: types of flip-flops and excitation table: T flip-flops are selected and the excitation table of the modulo-10 up down counter using T flip-flops is drawn as shown in fig.

The remaining minterms are don't cares ($\sum d(20,21,22,23,24,25,26,27,28,29,30,31)$) from the excitation table we can see that $T1=1$ and the expression for $T4, T3, T2$ are as follows.

$$T4 = \sum m(0,15,16,19) + d(20,21,22,23,24,25,26,27,28,29,30,31)$$

$$T3 = \sum m(7,15,16,8) + d(20,21,22,23,24,25,26,27,28,29,30,31)$$

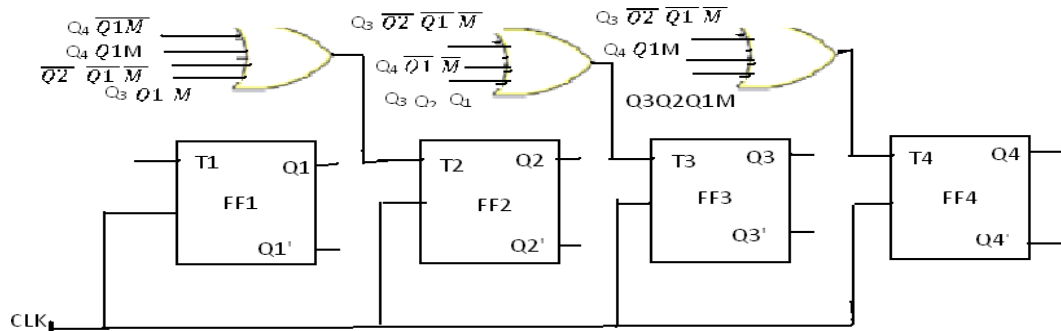
$$T2 = \sum m(3,4,7,8,11,12,15,16) + d(20,21,22,23,24,25,26,27,28,29,30,31)$$

PS					NS				required excitations			
Q4	Q3	Q2	Q1	mode	Q4	Q3	Q2	Q1	T4	T3	T2	T1
0	0	0	0	0	1	0	0	1	1	0	0	1
0	0	0	0	1	0	0	0	1	0	0	0	1
0	0	0	1	0	0	0	0	0	0	0	0	1
0	0	0	1	1	0	0	1	0	0	0	1	1
0	0	1	0	0	0	0	0	1	0	0	1	1
0	0	1	0	1	0	0	1	1	0	0	0	1
0	0	1	1	0	0	0	1	0	0	0	0	1
0	0	1	1	1	0	1	0	0	0	1	1	1
0	1	0	0	0	0	0	1	1	0	1	1	1
0	1	0	0	1	0	1	0	1	0	0	0	1
0	1	0	1	0	0	1	0	0	0	0	0	1
0	1	0	1	1	0	1	1	0	0	0	1	1
0	1	1	0	0	0	1	0	1	0	0	1	1
0	1	1	0	1	0	1	1	1	0	0	0	1
0	1	1	1	0	0	1	1	0	0	0	0	1
0	1	1	1	1	1	0	0	0	1	1	1	1
1	0	0	0	0	0	1	1	1	1	1	1	1
1	0	0	0	1	1	0	0	1	0	0	0	1
1	0	0	1	0	1	0	0	0	0	0	0	1
1	0	0	1	1	0	0	0	0	1	0	0	1

Step4: The minimal expression: since there are 4 state variables and a mode signal, we require 5 variable kmaps. 20 conditions of $Q_4Q_3Q_2Q_1M$ are valid and the remaining 12 combinations are invalid. So the entries for excitations corresponding to those invalid combinations are don't cares. Minimizing K-maps for T2 we get

$$T_2 = Q_4Q_1'M + Q_4'Q_1M + Q_2Q_1'M' + Q_3Q_1'M'$$

Step5: the logic diagram: the logic diagram based on the above equation is shown in fig.



Shift register counters:

One of the applications of shift register is that they can be arranged to form several types of counters. The most widely used shift register counter is ring counter as well as the twisted ring counter.

Ring counter: this is the simplest shift register counter. The basic ring counter using D flip-flops is shown in fig. the realization of this counter using JK FFs. The Q output of each stage is connected to the D flip-flop connected back to the ring counter.

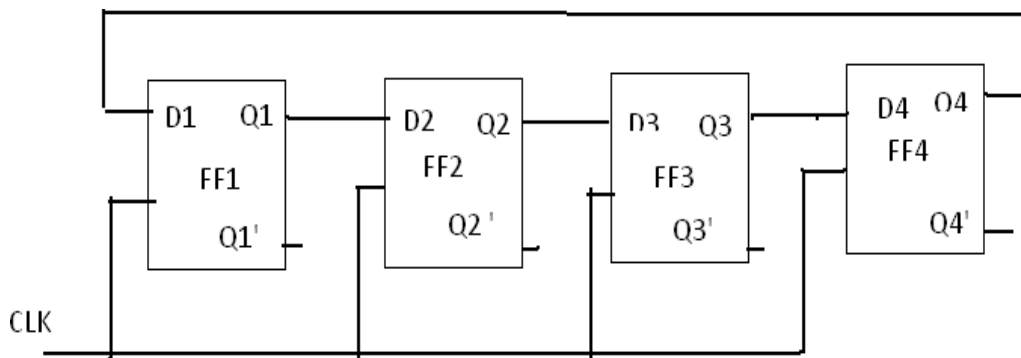
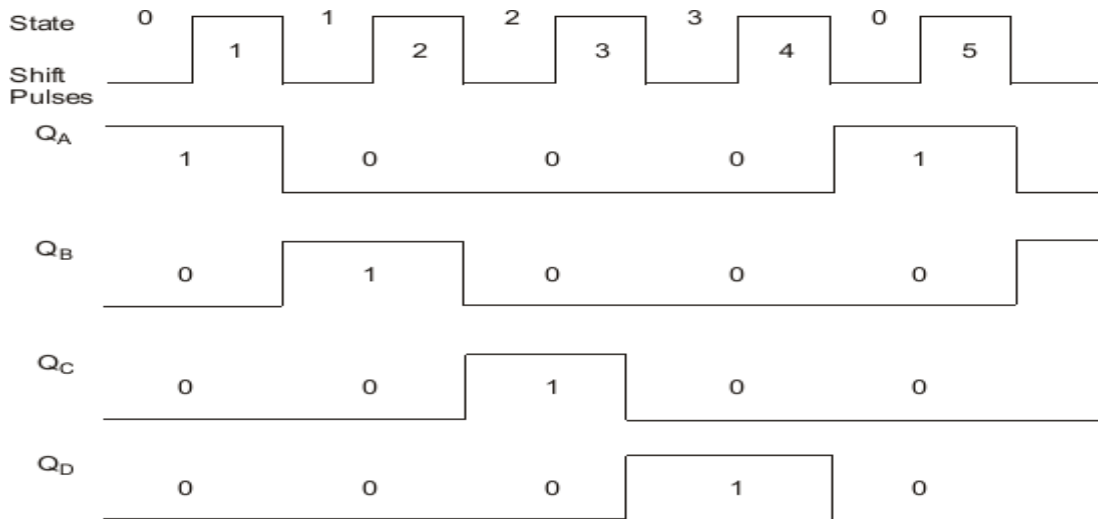


FIGURE: logic diagram of 4-bit ring counter using D flip-flops

Only a single 1 is in the register and is made to circulate around the register as long as clock pulses are applied. Initially the first FF is present to a 1. So, the initial state is 1000, i.e., $Q_1=1, Q_2=0, Q_3=0, Q_4=0$. After each clock pulse, the contents of the register are shifted to the right by one bit and Q_4 is shifted back to Q_1 . The sequence repeats after four clock pulses. The number

of distinct states in the ring counter, i.e., the mod of the ring counter is equal to number of FFs used in the counter. An n-bit ring counter can count only n bits, whereas n-bit ripple counter can count 2^n bits. So, the ring counter is uneconomical compared to a ripple counter but has advantage of requiring no decoder, since we can read the count by simply noting which FF is set. Since it is entirely a synchronous operation and requires no gates external FFs, it has the further advantage of being very fast.

Timing diagram:



Twisted Ring counter (Johnson counter):

This counter is obtained from a serial-in, serial-out shift register by providing feedback from the inverted output of the last FF to the D input of the first FF. The Q output of each is connected to the D input of the next stage, but the Q' output of the last stage is connected to the D input of the first stage, therefore, the name twisted ring counter. This feedback arrangement produces a unique sequence of states.

The logic diagram of a 4-bit Johnson counter using D FF is shown in fig. the realization of the same using J-K FFs is shown in fig.. The state diagram and the sequence table are shown in figure. The timing diagram of a Johnson counter is shown in figure.

Let initially all the FFs be reset, i.e., the state of the counter be 0000. After each clock pulse, the level of Q1 is shifted to Q2, the level of Q2 to Q3, Q3 to Q4 and the level of Q4' to Q1 and the sequences given in fig.

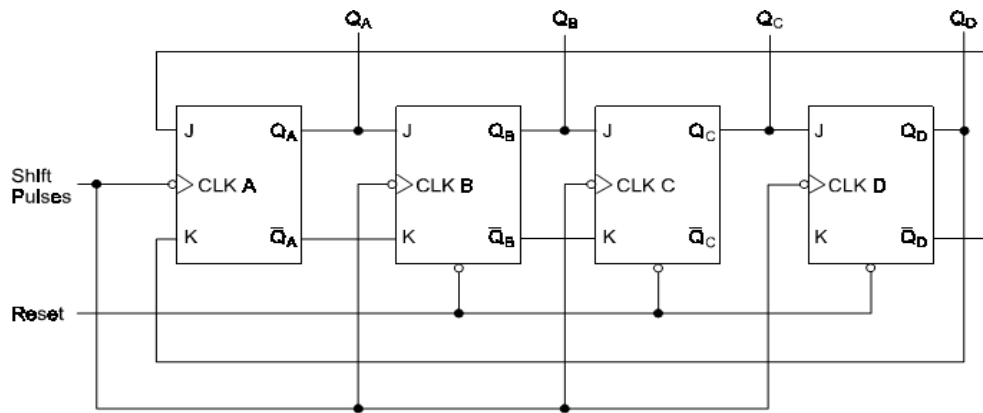


Figure: Johnson counter with JK flip-flops

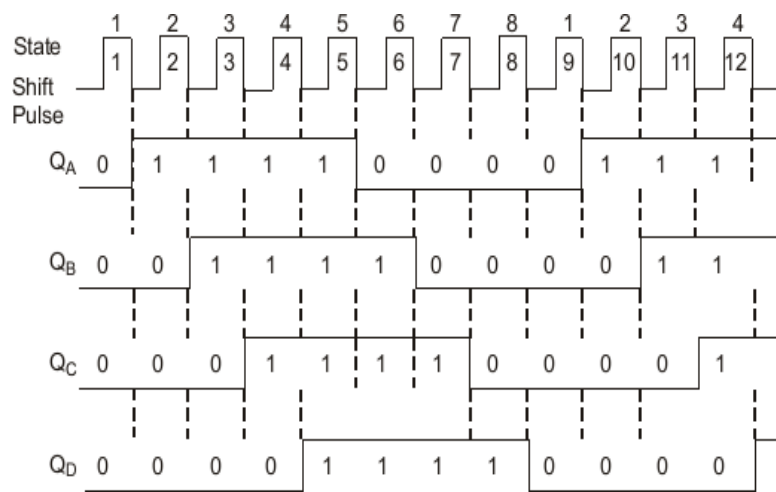


Figure: timing diagram

UNIT-IV 8085 MICROPROCESSOR

1. INTRODUCTION TO MICROPROCESSOR AND MICROCOMPUTER ARCHITECTURE:

A *microprocessor* is a programmable electronics chip that has computing and decision making capabilities similar to central processing unit of a computer. Any microprocessor-based systems having limited number of resources are called *microcomputers*. Nowadays, microprocessor can be seen in almost all types of electronics devices like mobile phones, printers, washing machines etc. Microprocessors are also used in advanced applications like radars, satellites and flights. Due to the rapid advancements in electronic industry and large scale integration of devices results in a significant cost reduction and increase application of microprocessors and their derivatives.

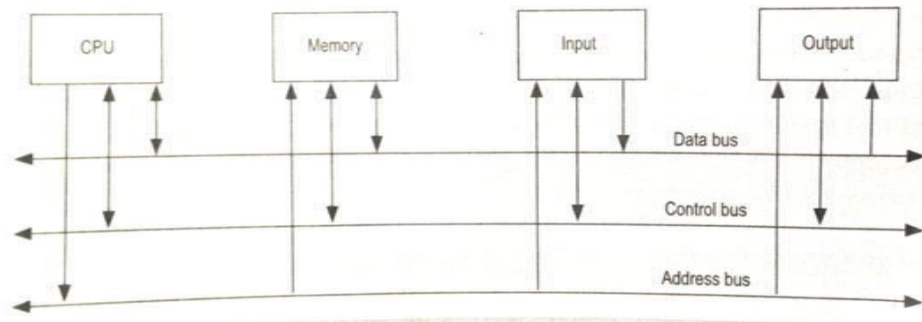


Fig.1 Microprocessor-based system

- **Bit:** A bit is a single binary digit.
- **Word:** A word refers to the basic data size or bit size that can be processed by the arithmetic and logic unit of the processor. A 16-bit binary number is called a word in a 16-bit processor.
- **Bus:** A bus is a group of wires/lines that carry similar information.
- **System Bus:** The system bus is a group of wires/lines used for communication between the microprocessor and peripherals.
- **Memory Word:** The number of bits that can be stored in a register or memory element is called a memory word.
- **Address Bus:** It carries the address, which is a unique binary pattern used to identify a memory location or an I/O port. For example, an eight bit address bus has eight lines and thus it can address $2^8 = 256$ different locations. The locations in hexadecimal format can be written as 00H – FFH.
- **Data Bus:** The data bus is used to transfer data between memory and processor or between I/O device and processor. For example, an 8-bit processor will generally have an 8-bit data bus and a 16-bit processor will have 16-bit data bus.
- **Control Bus:** The control bus carry control signals, which consists of signals for selection of memory or I/O device from the given address, direction of data transfer and synchronization of data transfer in case of slow devices.

A typical microprocessor consists of arithmetic and logic unit (ALU) in association with control unit to process the instruction execution. Almost all the microprocessors are based on the principle of store-program concept. In *store-program concept*, programs or instructions are sequentially stored in the memory locations that are to be executed. To do any task using a microprocessor, it is to be programmed by the user. So the programmer must have idea about its internal resources, features and supported instructions. Each microprocessor has a set of instructions, a list which is provided by the microprocessor manufacturer. The instruction set of a microprocessor is provided in two forms: *binary machine code and mnemonics*.

Microprocessor communicates and operates in binary numbers 0 and 1. The set of instructions in the form of binary patterns is called a *machine language* and it is difficult for us to understand. Therefore, the binary patterns are given abbreviated names, called mnemonics, which forms the *assembly language*. The conversion of assembly-level language into binary machine-level language is done by using an application called *assembler*.

Technology Used:

The semiconductor manufacturing technologies used for chips are:

- Transistor-Transistor Logic (TTL)
- Emitter Coupled Logic (ECL)
- Complementary Metal-Oxide Semiconductor (CMOS)

Classification of Microprocessors:

Based on their specification, application and architecture microprocessors are classified.

Based on size of data bus:

- 4-bit microprocessor
- 8-bit microprocessor
- 16-bit microprocessor
- 32-bit microprocessor

Based on application:

- General-purpose microprocessor- used in general computer system and can be used by programmer for any application. Examples, 8085 to Intel Pentium.
- Microcontroller- microprocessor with built-in memory and ports and can be programmed for any generic control application. Example, 8051.
- Special-purpose processors- designed to handle special functions required for an application. Examples, digital signal processors and application-specific integrated circuit (ASIC) chips.

Based on architecture:

- Reduced Instruction Set Computer (RISC) processors
- Complex Instruction Set Computer (CISC) processors

2. 8085 MICROPROCESSOR ARCHITECTURE

The 8085 microprocessor is an 8-bit processor available as a 40-pin IC package and uses +5 V for power. It can run at a maximum frequency of 3 MHz. Its data bus width is 8-bit and address bus width is 16-bit, thus it can address $2^{16} = 64$ KB of memory. The internal architecture of 8085 is shown in Fig. 2.

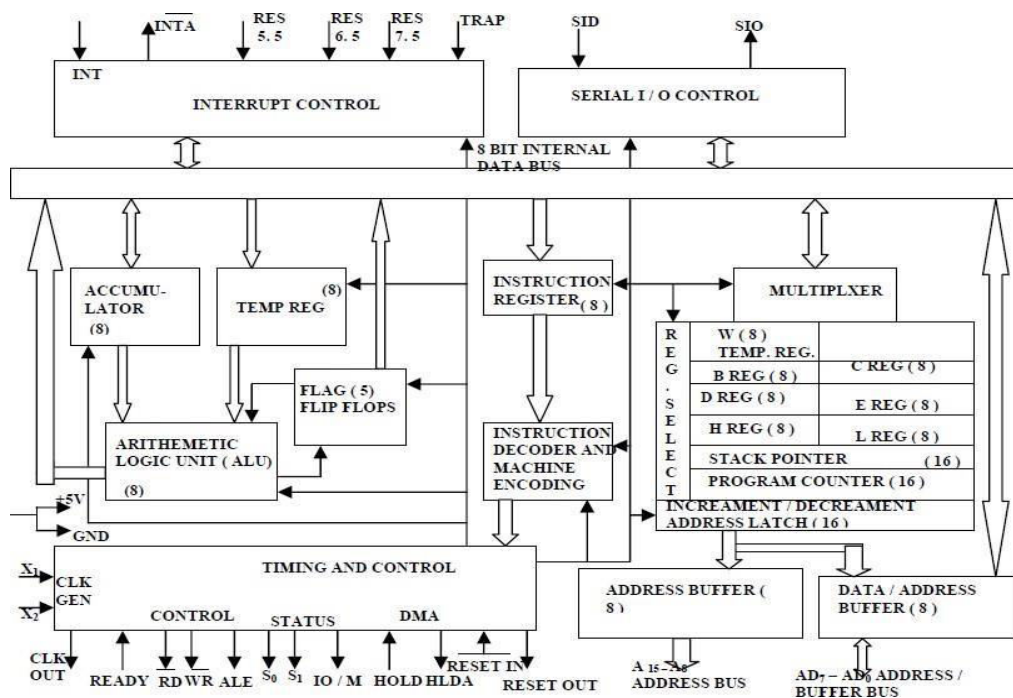


Fig. 2 Internal Architecture of 8085

Arithmetic and Logic Unit

The ALU performs the actual numerical and logical operations such as Addition (ADD), Subtraction (SUB), AND, OR etc. It uses data from memory and from Accumulator to perform operations. The results of the arithmetic and logical operations are stored in the accumulator.

Registers

The 8085 includes six registers, one accumulator and one flag register, as shown in Fig. 3. In addition, it has two 16-bit registers: stack pointer and program counter. They are briefly described as follows.

The 8085 has six general-purpose registers to store 8-bit data; these are identified as B, C, D, E, H and L. they can be combined as register pairs - BC, DE and HL to perform some

16-bit operations. The programmer can use these registers to store or copy data into the register by using data copy instructions.

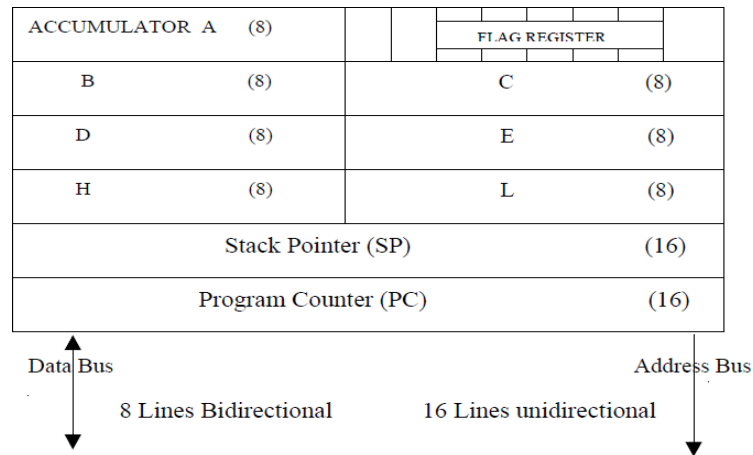


Fig. 3 Register organisation

Accumulator

The accumulator is an 8-bit register that is a part of ALU. This register is used to store 8-bit data and to perform arithmetic and logical operations. The result of an operation is stored in the accumulator. The accumulator is also identified as register A.

Flag register

The ALU includes five flip-flops, which are set or reset after an operation according to data condition of the result in the accumulator and other registers. They are called Zero (Z), Carry (CY), Sign (S), Parity (P) and Auxiliary Carry (AC) flags. Their bit positions in the flag register are shown in Fig. 4. The microprocessor uses these flags to test data conditions.

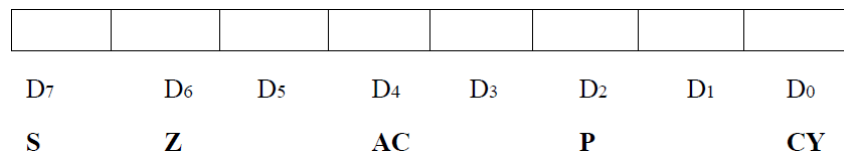


Fig. 4 Flag register

For example, after an addition of two numbers, if the result in the accumulator is larger than 8-bit, the flip-flop uses to indicate a carry by setting CY flag to 1. When an arithmetic operation results in zero, Z flag is set to 1. The S flag is just a copy of the bit D7 of the accumulator. A negative number has a 1 in bit D7 and a positive number has a 0 in 2's complement representation. The AC flag is set to 1, when a carry result from bit D3 and passes to bit D4. The P flag is set to 1, when the result in accumulator contains even number of 1s.

Program Counter (PC)

This 16-bit register deals with sequencing the execution of instructions. This register is a memory pointer. The microprocessor uses this register to sequence the execution of the instructions. The function of the program counter is to point to the memory address from which the next byte is to be fetched. When a byte is being fetched, the program counter is automatically incremented by one to point to the next memory location.

Stack Pointer (SP)

The stack pointer is also a 16-bit register, used as a memory pointer. It points to a memory location in R/W memory, called stack. The beginning of the stack is defined by loading 16-bit address in the stack pointer.

Instruction Register/Decoder

It is an 8-bit register that temporarily stores the current instruction of a program. Latest instruction sent here from memory prior to execution. Decoder then takes instruction and decodes or interprets the instruction. Decoded instruction then passed to next stage.

Control Unit

Generates signals on data bus, address bus and control bus within microprocessor to carry out the instruction, which has been decoded. Typical buses and their timing are described as follows:

- *Data Bus*: Data bus carries data in binary form between microprocessor and other external units such as memory. It is used to transmit data i.e. information, results of arithmetic etc between memory and the microprocessor. Data bus is bidirectional in nature. The data bus width of 8085 microprocessor is 8-bit i.e. 2^8 combination of binary digits and are typically identified as D0 – D7. Thus size of the data bus determines what arithmetic can be done. If only 8-bit wide then largest number is 11111111 (255 in decimal). Therefore, larger numbers have to be broken down into chunks of 255. This slows microprocessor.
- *Address Bus*: The address bus carries addresses and is one way bus from microprocessor to the memory or other devices. 8085 microprocessor contain 16-bit address bus and are generally identified as A0 - A15. The higher order address lines (A8 – A15) are unidirectional and the lower order lines (A0 – A7) are multiplexed (time-shared) with the eight data bits (D0 – D7) and hence, they are bidirectional.
- *Control Bus*: Control bus are various lines which have specific functions for coordinating and controlling microprocessor operations. The control bus carries control signals partly unidirectional and partly bidirectional. The following control and status signals are used by 8085 processor:
 - I. ALE (output): Address Latch Enable is a pulse that is provided when an address appears on the AD0 – AD7 lines, after which it becomes 0.

- II. \overline{RD} (active low output): The Read signal indicates that data are being read from the selected I/O or memory device and that they are available on the data bus.
- III. \overline{WR} (active low output): The Write signal indicates that data on the data bus are to be written into a selected memory or I/O location.
- IV. $\overline{IO/\overline{M}}$ (output): It is a signal that distinguished between a memory operation and an I/O operation. When $\overline{IO/\overline{M}} = 0$ it is a memory operation and $\overline{IO/\overline{M}} = 1$ it is an I/O operation.
- V. S1 and S0 (output): These are status signals used to specify the type of operation being performed; they are listed in Table 1.

Table 1 Status signals and associated operations

S1	S0	States
0	0	Halt
0	1	Write
1	0	Read
1	1	Fetch

The schematic representation of the 8085 bus structure is as shown in Fig. 5. The microprocessor performs primarily four operations:

- I. Memory Read: Reads data (or instruction) from memory.
- II. Memory Write: Writes data (or instruction) into memory.
- III. I/O Read: Accepts data from input device.
- IV. I/O Write: Sends data to output device.

The 8085 processor performs these functions using address bus, data bus and control bus as shown in Fig. 5.

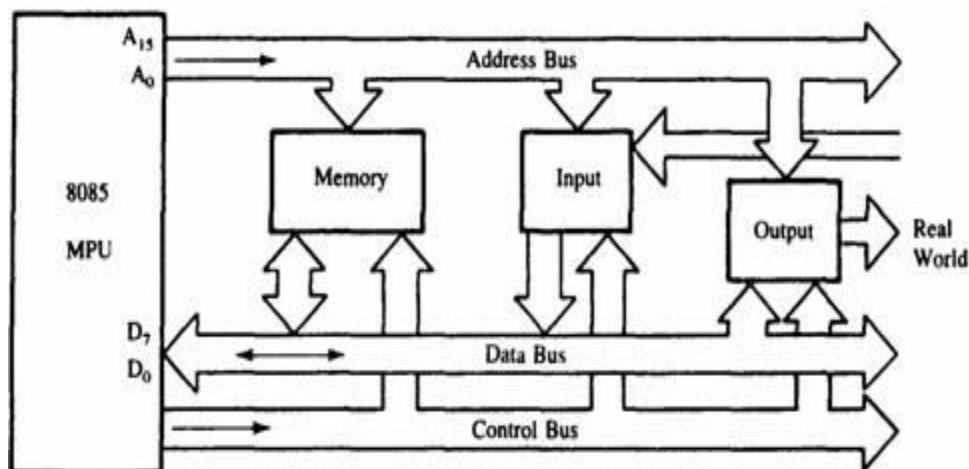


Fig. 5 The 8085 bus structure

3. 8085 PIN DESCRIPTION

Properties:

- It is a 8-bit microprocessor
- Manufactured with N-MOS technology
- 40 pin IC package
- It has 16-bit address bus and thus has $2^{16} = 64$ KB addressing capability.
- Operate with 3 MHz single-phase clock
- +5 V single power supply

The logic pin layout and signal groups of the 8085 microprocessor are shown in Fig. 6. All the signals are classified into six groups:

- Address bus
- Data bus
- Control & status signals
- Power supply and frequency signals
- Externally initiated signals
- Serial I/O signals

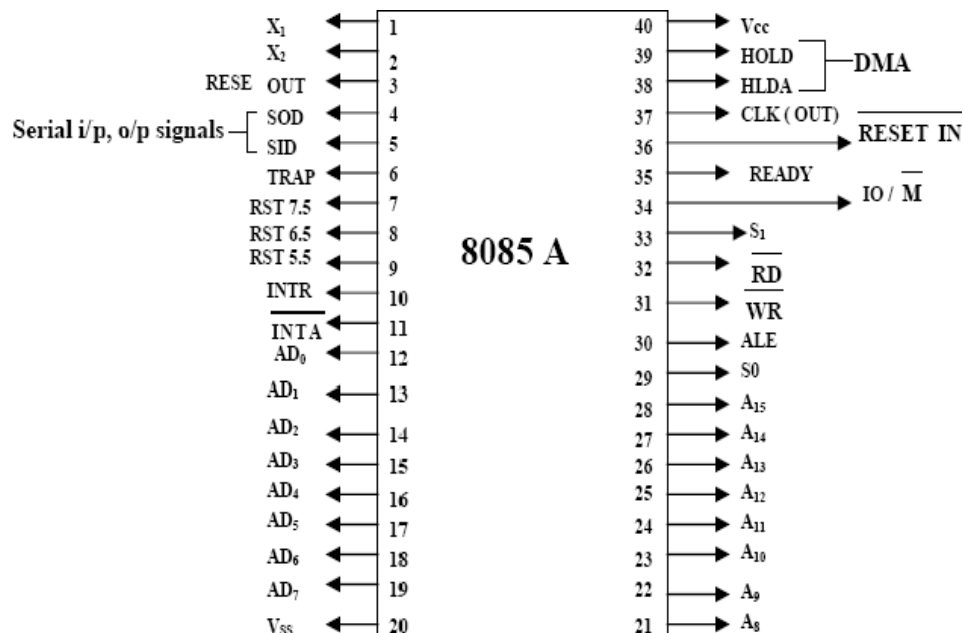


Fig. 6 8085 microprocessor pin layout and signal groups

Address and Data Buses:

- A₈ – A₁₅ (output, 3-state): Most significant eight bits of memory addresses and the eight bits of the I/O addresses. These lines enter into tri-state high impedance state during HOLD and HALT modes.
- AD₀ – AD₇ (input/output, 3-state): Lower significant bits of memory addresses and

the eight bits of the I/O addresses during first clock cycle. Behaves as data bus

during third and fourth clock cycle. These lines enter into tri-state high impedance state during HOLD and HALT modes.

Control & Status Signals:

- $\overline{\text{ALE}}$: Address latch enable
- $\overline{\text{RD}}$: Read control signal.
- $\overline{\text{WR}}$: Write control signal.
- IO/M, S1 and S0 : Status signals.

Power Supply & Clock Frequency:

- Vcc: +5 V power supply
- Vss: Ground reference
- X1, X2: A crystal having frequency of 6 MHz is connected at these two pins
- CLK: Clock output

Externally Initiated and Interrupt Signals:

- $\overline{\text{RESET IN}}$: When the signal on this pin is low, the PC is set to 0, the buses are tri-stated and the processor is reset.
- RESET OUT: This signal indicates that the processor is being reset. The signal can be used to reset other devices.
- READY: When this signal is low, the processor waits for an integral number of clock cycles until it goes high.
- HOLD: This signal indicates that a peripheral like DMA (direct memory access) controller is requesting the use of address and data bus.
- HLDA: This signal acknowledges the HOLD request.
- $\overline{\text{INTR}}$: Interrupt request is a general-purpose interrupt.
- $\overline{\text{INTA}}$: This is used to acknowledge an interrupt.
- RST 7.5, RST 6.5, RST 5.5 – restart interrupt: These are vectored interrupts and have highest priority than INTR interrupt.
- TRAP: This is a non-maskable interrupt and has the highest priority.

Serial I/O Signals:

- SID: Serial input signal. Bit on this line is loaded to D7 bit of register A using RIM instruction.
- SOD: Serial output signal. Output SOD is set or reset by using SIM instruction.
-

4. INSTRUCTION SET AND EXECUTION IN 8085

Based on the design of the ALU and decoding unit, the microprocessor manufacturer provides instruction set for every microprocessor. The instruction set consists of both machine code and mnemonics.

An instruction is a binary pattern designed inside a microprocessor to perform a specific function. The entire group of instructions that a microprocessor supports is called instruction set. Microprocessor instructions can be classified based on the parameters such functionality, length and operand addressing.

Classification based on functionality:

- I. Data transfer operations: This group of instructions copies data from source to destination. The content of the source is not altered.
- II. Arithmetic operations: Instructions of this group perform operations like addition, subtraction, increment & decrement. One of the data used in arithmetic operation is stored in accumulator and the result is also stored in accumulator.
- III. Logical operations: Logical operations include AND, OR, EXOR, NOT. The operations like AND, OR and EXOR uses two operands, one is stored in accumulator and other can be any register or memory location. The result is stored in accumulator. NOT operation requires single operand, which is stored in accumulator.
- IV. Branching operations: Instructions in this group can be used to transfer program sequence from one memory location to another either conditionally or unconditionally.
- V. Machine control operations: Instruction in this group control execution of other instructions and control operations like interrupt, halt etc.

Classification based on length:

- I. One-byte instructions: Instruction having one byte in machine code. Examples are depicted in Table 2.
- I. Two-byte instructions: Instruction having two byte in machine code. Examples are depicted in Table 3
- II. Three-byte instructions: Instruction having three byte in machine code. Examples are depicted in Table 4.

Table 2 Examples of one byte instructions

Opcode	Operand	Machine code/Hex code
MOV	A, B	78
ADD	M	86

Table 3 Examples of two byte instructions

Opcode	Operand	Machine code/Hex code	Byte description
MVI	A, 7FH	3E	First byte
		7F	Second byte
ADI	0FH	C6	First byte
		0F	Second byte

Table 4 Examples of three byte instructions

Opcode	Operand	Machine code/Hex code	Byte description
JMP	9050H	C3	First byte
		50	Second byte
		90	Third byte
LDA	8850H	3A	First byte
		50	Second byte
		88	Third byte

Addressing Modes in Instructions:

The process of specifying the data to be operated on by the instruction is called addressing. The various formats for specifying operands are called addressing modes. The 8085 has the following five types of addressing:

- I. Immediate addressing
- II. Memory direct addressing
- III. Register direct addressing
- IV. Indirect addressing
- V. Implicit addressing

Immediate Addressing:

In this mode, the operand given in the instruction - a byte or word – transfers to the destination register or memory location.

Ex: MVI A, 9AH

- The operand is a part of the instruction.
- The operand is stored in the register mentioned in the instruction.

Memory Direct Addressing:

Memory direct addressing moves a byte or word between a memory location and register. The memory location address is given in the instruction.

Ex: LDA 850FH

This instruction is used to load the content of memory address 850FH in the accumulator.

Register Direct Addressing:

Register direct addressing transfer a copy of a byte or word from source register to destination register.

Ex: MOV B, C

It copies the content of register C to register B.

Indirect Addressing:

Indirect addressing transfers a byte or word between a register and a memory location.

Ex: MOV A, M

Here the data is in the memory location pointed to by the contents of HL pair. The data is moved to the accumulator.

Implicit Addressing

In this addressing mode the data itself specifies the data to be operated upon.

Ex: CMA

The instruction complements the content of the accumulator. No specific data or operand is mentioned in the instruction.

5. INSTRUCTION SET OF 8085

Data Transfer Instructions:

Opcode	Operand	Description
Copy from source to destination MOV	Rd, Rs M, Rs Rd, M	This instruction copies the contents of the source register into the destination register; the contents of the source register are not altered. If one of the operands is a memory location, its location is specified by the contents of the HL registers. Example: MOV B, C or MOV B, M
Move immediate 8-bit MVI	Rd, data M, data	The 8-bit data is stored in the destination register or memory. If the operand is a memory location, its location is specified by the contents of the HL registers. Example: MVI B, 57 or MVI M, 57
Load accumulator LDA	16-bit address	The contents of a memory location, specified by a 16-bit address in the operand, are copied to the accumulator. The contents of the source are not altered. Example: LDA 2034 or LDA XYZ
Load accumulator indirect LDAX	B/D Reg. pair	The contents of the designated register pair point to a memory location. This instruction copies the contents of that memory location into the accumulator. The contents of either the register pair or the memory location are not altered. Example: LDAX B
Load register pair immediate LXI	Reg. pair, 16-bit data	The instruction loads 16-bit data in the register pair designated in the operand. Example: LXI H, 2034
Load H and L registers direct LHLD	16-bit address	The instruction copies the contents of the memory location pointed out by the 16-bit address into register L and copies the contents of the next memory location into register H. The contents of source memory locations are not altered. Example: LHLD 2040

Store accumulator direct
STA 16-bit address

The contents of the accumulator are copied into the memory location specified by the operand. This is a 3-byte instruction, the second byte specifies the low-order address and the third byte specifies the high-order address.
Example: STA 4350 or STA XYZ

Store accumulator indirect
STAX Reg. pair

The contents of the accumulator are copied into the memory location specified by the contents of the operand (register pair). The contents of the accumulator are not altered.
Example: STAX B

Store H and L registers direct
SHLD 16-bit address

The contents of register L are stored into the memory location specified by the 16-bit address in the operand and the contents of H register are stored into the next memory location by incrementing the operand. The contents of registers HL are not altered. This is a 3-byte instruction, the second byte specifies the low-order address and the third byte specifies the high-order address.
Example: SHLD 2470

Exchange H and L with D and E
XCHG none

The contents of register H are exchanged with the contents of register D, and the contents of register L are exchanged with the contents of register E.
Example: XCHG

Copy H and L registers to the stack pointer
SPHL none

The instruction loads the contents of the H and L registers into the stack pointer register, the contents of the H register provide the high-order address and the contents of the L register provide the low-order address. The contents of the H and L registers are not altered.
Example: SPHL

Exchange H and L with top of stack
XTHL none

The contents of the L register are exchanged with the stack location pointed out by the contents of the stack pointer register. The contents of the H register are exchanged with the next stack location (SP+1); however, the contents of the stack pointer register are not altered.
Example: XTHL

Push register pair onto stack
PUSH Reg. pair

The contents of the register pair designated in the operand are copied onto the stack in the following sequence. The stack pointer register is decremented and the contents of the high-order register (B, D, H, A) are copied into that location. The stack pointer register is decremented again and the contents of the low-order register (C, E, L, flags) are copied to that location.
Example: PUSH B or PUSH A

Pop off stack to register pair
POP Reg. pair

The contents of the memory location pointed out by the stack pointer register are copied to the low-order register (C, E, L, status flags) of the operand. The stack pointer is incremented by 1 and the contents of that memory location are copied to the high-order register (B, D, H, A) of the operand. The stack pointer register is again incremented by 1.
Example: POP H or POP A

Output data from accumulator to a port with 8-bit address
OUT 8-bit port address

The contents of the accumulator are copied into the I/O port specified by the operand.
Example: OUT 87

Input data to accumulator from a port with 8-bit address
IN 8-bit port address

The contents of the input port designated in the operand are read and loaded into the accumulator.
Example: IN 82

Arithmetic Instructions:

Opcode	Operand	Description
Add register or memory to accumulator		
ADD	R M	The contents of the operand (register or memory) are added to the contents of the accumulator and the result is stored in the accumulator. If the operand is a memory location, its location is specified by the contents of the HL registers. All flags are modified to reflect the result of the addition. Example: ADD B or ADD M
Add register to accumulator with carry		
ADC	R M	The contents of the operand (register or memory) and the Carry flag are added to the contents of the accumulator and the result is stored in the accumulator. If the operand is a memory location, its location is specified by the contents of the HL registers. All flags are modified to reflect the result of the addition. Example: ADC B or ADC M
Add immediate to accumulator		
ADI	8-bit data	The 8-bit data (operand) is added to the contents of the accumulator and the result is stored in the accumulator. All flags are modified to reflect the result of the addition. Example: ADI 45
Add immediate to accumulator with carry		
ACI	8-bit data	The 8-bit data (operand) and the Carry flag are added to the contents of the accumulator and the result is stored in the accumulator. All flags are modified to reflect the result of the addition. Example: ACI 45
Add register pair to H and L registers		
DAD	Reg. pair	The 16-bit contents of the specified register pair are added to the contents of the HL register and the sum is stored in the HL register. The contents of the source register pair are not altered. If the result is larger than 16 bits, the CY flag is set. No other flags are affected. Example: DAD H

Subtract register or memory from accumulator

SUB R
 M

The contents of the operand (register or memory) are subtracted from the contents of the accumulator, and the result is stored in the accumulator. If the operand is a memory location, its location is specified by the contents of the HL registers. All flags are modified to reflect the result of the subtraction.
Example: SUB B or SUB M

Subtract source and borrow from accumulator

SBB R
 M

The contents of the operand (register or memory) and the Borrow flag are subtracted from the contents of the accumulator and the result is placed in the accumulator. If the operand is a memory location, its location is specified by the contents of the HL registers. All flags are modified to reflect the result of the subtraction.
Example: SBB B or SBB M

Subtract immediate from accumulator

SUI 8-bit data

The 8-bit data (operand) is subtracted from the contents of the accumulator and the result is stored in the accumulator. All flags are modified to reflect the result of the subtraction.
Example: SUI 45

Subtract immediate from accumulator with borrow

SBI 8-bit data

The 8-bit data (operand) and the Borrow flag are subtracted from the contents of the accumulator and the result is stored in the accumulator. All flags are modified to reflect the result of the subtraction.
Example: SBI 45

Increment register or memory by 1

INR R
 M

The contents of the designated register or memory) are incremented by 1 and the result is stored in the same place. If the operand is a memory location, its location is specified by the contents of the HL registers.
Example: INR B or INR M

Increment register pair by 1

INX R

The contents of the designated register pair are incremented by 1 and the result is stored in the same place.
Example: INX H

Decrement register or memory by 1

DCR R
 M

The contents of the designated register or memory are decremented by 1 and the result is stored in the same place. If the operand is a memory location, its location is specified by the contents of the HL registers.
Example: DCR B or DCR M

Decrement register pair by 1

DCX R

The contents of the designated register pair are decremented by 1 and the result is stored in the same place.
Example: DCX H

Decimal adjust accumulator

DAA none

The contents of the accumulator are changed from a binary value to two 4-bit binary coded decimal (BCD) digits. This is the only instruction that uses the auxiliary flag to perform the binary to BCD conversion, and the conversion procedure is described below. S, Z, AC, P, CY flags are altered to reflect the results of the operation.

If the value of the low-order 4-bits in the accumulator is greater than 9 or if AC flag is set, the instruction adds 6 to the low-order four bits.

If the value of the high-order 4-bits in the accumulator is greater than 9 or if the Carry flag is set, the instruction adds 6 to the high-order four bits.

Example: DAA

BRANCHING INSTRUCTIONS

Opcode Operand Description

Jump unconditionally

JMP 16-bit address

The program sequence is transferred to the memory location specified by the 16-bit address given in the operand.
Example: JMP 2034 or JMP XYZ

Jump conditionally

Operand: 16-bit address

The program sequence is transferred to the memory location specified by the 16-bit address given in the operand based on the specified flag of the PSW as described below.
Example: JZ 2034 or JZ XYZ

Opcode	Description	Flag Status
JC	Jump on Carry	CY = 1
JNC	Jump on no Carry	CY = 0
JP	Jump on positive	S = 0
JM	Jump on minus	S = 1
JZ	Jump on zero	Z = 1
JNZ	Jump on no zero	Z = 0
JPE	Jump on parity even	P = 1
JPO	Jump on parity odd	P = 0

Unconditional subroutine call

CALL 16-bit address

The program sequence is transferred to the memory location specified by the 16-bit address given in the operand. Before the transfer, the address of the next instruction after CALL (the contents of the program counter) is pushed onto the stack. Example: CALL 2034 or CALL XYZ

Call conditionally

Operand: 16-bit address

The program sequence is transferred to the memory location specified by the 16-bit address given in the operand based on the specified flag of the PSW as described below. Before the transfer, the address of the next instruction after the call (the contents of the program counter) is pushed onto the stack. Example: CZ 2034 or CZ XYZ

Opcode	Description	Flag Status
CC	Call on Carry	CY = 1
CNC	Call on no Carry	CY = 0
CP	Call on positive	S = 0
CM	Call on minus	S = 1
CZ	Call on zero	Z = 1
CNZ	Call on no zero	Z = 0
CPE	Call on parity even	P = 1
CPO	Call on parity odd	P = 0

Return from subroutine unconditionally

RET none

The program sequence is transferred from the subroutine to the calling program. The two bytes from the top of the stack are copied into the program counter, and program execution begins at the new address. Example: RET

Return from subroutine conditionally

Operand: none

The program sequence is transferred from the subroutine to the calling program based on the specified flag of the PSW as described below. The two bytes from the top of the stack are copied into the program counter, and program execution begins at the new address. Example: RZ

Opcode	Description	Flag Status
RC	Return on Carry	CY = 1
RNC	Return on no Carry	CY = 0
RP	Return on positive	S = 0
RM	Return on minus	S = 1
RZ	Return on zero	Z = 1
RNZ	Return on no zero	Z = 0
RPE	Return on parity even	P = 1
RPO	Return on parity odd	P = 0

Load program counter with HL contents

PCHL none The contents of registers H and L are copied into the program counter. The contents of H are placed as the high-order byte and the contents of L as the low-order byte.
Example: PCHL

Restart

RST 0-7 The RST instruction is equivalent to a 1-byte call instruction to one of eight memory locations depending upon the number. The instructions are generally used in conjunction with interrupts and inserted using external hardware. However these can be used as software instructions in a program to transfer program execution to one of the eight locations. The addresses are:

Instruction	Restart Address
RST 0	0000H
RST 1	0008H
RST 2	0010H
RST 3	0018H
RST 4	0020H
RST 5	0028H
RST 6	0030H
RST 7	0038H

The 8085 has four additional interrupts and these interrupts generate RST instructions internally and thus do not require any external hardware. These instructions and their Restart addresses are:

Interrupt	Restart Address
TRAP	0024H
RST 5.5	002CH
RST 6.5	0034H
RST 7.5	003CH

LOGICAL INSTRUCTIONS

Opcode Operand Description

Compare register or memory with accumulator

CMP R The contents of the operand (register or memory) are compared with the contents of the accumulator. Both contents are preserved. The result of the comparison is shown by setting the flags of the PSW as follows:
if (A) < (reg/mem): carry flag is set, s=1
if (A) = (reg/mem): zero flag is set, s=0
if (A) > (reg/mem): carry and zero flags are reset, s=0
Example: CMP B or CMP M

Compare immediate with accumulator

CPI 8-bit data The second byte (8-bit data) is compared with the contents of the accumulator. The values being compared remain unchanged. The result of the comparison is shown by setting the flags of the PSW as follows:
if (A) < data: carry flag is set, s=1
if (A) = data: zero flag is set, s=0
if (A) > data: carry and zero flags are reset, s=0
Example: CPI 89

Logical AND register or memory with accumulator

ANA R The contents of the accumulator are logically ANDed with the contents of the operand (register or memory), and the result is placed in the accumulator. If the operand is a memory location, its address is specified by the contents of HL registers. S, Z, P are modified to reflect the result of the operation. CY is reset. AC is set.
Example: ANA B or ANA M

Logical AND immediate with accumulator

ANI 8-bit data The contents of the accumulator are logically ANDed with the 8-bit data (operand) and the result is placed in the accumulator. S, Z, P are modified to reflect the result of the operation. CY is reset. AC is set.
Example: ANI 86

Exclusive OR register or memory with accumulator

XRA R
 M

The contents of the accumulator are Exclusive ORed with the contents of the operand (register or memory), and the result is placed in the accumulator. If the operand is a memory location, its address is specified by the contents of HL registers. S, Z, P are modified to reflect the result of the operation. CY and AC are reset.
Example: XRA B or XRA M

Exclusive OR immediate with accumulator

XRI 8-bit data

The contents of the accumulator are Exclusive ORed with the 8-bit data (operand) and the result is placed in the accumulator. S, Z, P are modified to reflect the result of the operation. CY and AC are reset.
Example: XRI 86

Logical OR register or memory with accumulator

ORA R
 M

The contents of the accumulator are logically ORed with the contents of the operand (register or memory), and the result is placed in the accumulator. If the operand is a memory location, its address is specified by the contents of HL registers. S, Z, P are modified to reflect the result of the operation. CY and AC are reset.
Example: ORA B or ORA M

Logical OR immediate with accumulator

ORI 8-bit data

The contents of the accumulator are logically ORed with the 8-bit data (operand) and the result is placed in the accumulator. S, Z, P are modified to reflect the result of the operation. CY and AC are reset.
Example: ORI 86

Rotate accumulator left

RLC none

Each binary bit of the accumulator is rotated left by one position. Bit D7 is placed in the position of D0 as well as in the Carry flag. CY is modified according to bit D7. S, Z, P, AC are not affected.
Example: RLC

Rotate accumulator right

RRC none

Each binary bit of the accumulator is rotated right by one position. Bit D0 is placed in the position of D7 as well as in the Carry flag. CY is modified according to bit D0. S, Z, P, AC are not affected.
Example: RRC

Rotate accumulator left through carry		
RAL	none	Each binary bit of the accumulator is rotated left by one position through the Carry flag. Bit D7 is placed in the Carry flag, and the Carry flag is placed in the least significant position D0. CY is modified according to bit D7. S, Z, P, AC are not affected. Example: RAL
Rotate accumulator right through carry		
RAR	none	Each binary bit of the accumulator is rotated right by one position through the Carry flag. Bit D0 is placed in the Carry flag, and the Carry flag is placed in the most significant position D7. CY is modified according to bit D0. S, Z, P, AC are not affected. Example: RAR
Complement accumulator		
CMA	none	The contents of the accumulator are complemented. No flags are affected. Example: CMA
Complement carry		
CMC	none	The Carry flag is complemented. No other flags are affected. Example: CMC
Set Carry		
STC	none	The Carry flag is set to 1. No other flags are affected. Example: STC

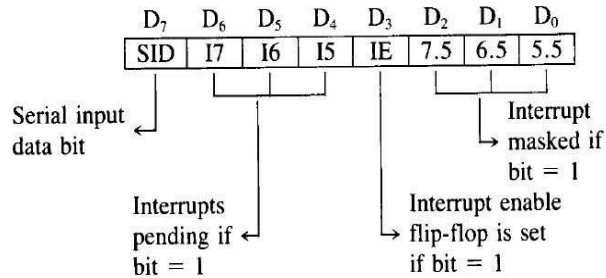
CONTROL INSTRUCTIONS

Opcode	Operand	Description
No operation		
NOP	none	No operation is performed. The instruction is fetched and decoded. However no operation is executed. Example: NOP
Halt and enter wait state		
HLT	none	The CPU finishes executing the current instruction and halts any further execution. An interrupt or reset is necessary to exit from the halt state. Example: HLT
Disable interrupts		
DI	none	The interrupt enable flip-flop is reset and all the interrupts except the TRAP are disabled. No flags are affected. Example: DI
Enable interrupts		
EI	none	The interrupt enable flip-flop is set and all interrupts are enabled. No flags are affected. After a system reset or the acknowledgement of an interrupt, the interrupt enable flip-flop is reset, thus disabling the interrupts. This instruction is necessary to reenable the interrupts (except TRAP). Example: EI

Read interrupt mask
RIM none

This is a multipurpose instruction used to read the status of interrupts 7.5, 6.5, 5.5 and read serial data input bit. The instruction loads eight bits in the accumulator with the following interpretations.

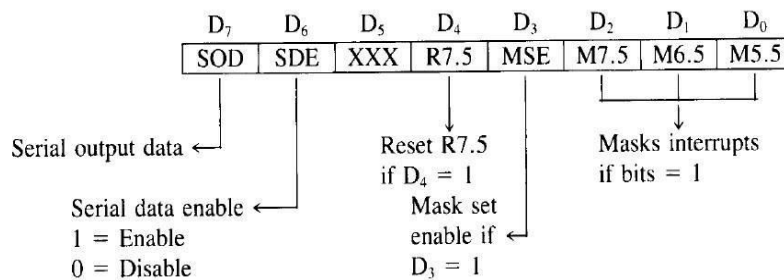
Example: RIM



Set interrupt mask
SIM none

This is a multipurpose instruction and used to implement the 8085 interrupts 7.5, 6.5, 5.5, and serial data output. The instruction interprets the accumulator contents as follows.

Example: SIM



- SOD—Serial Output Data: Bit D₇ of the accumulator is latched into the SOD output line and made available to a serial peripheral if bit D₆ = 1.
- SDE—Serial Data Enable: If this bit = 1, it enables the serial output. To implement serial output, this bit needs to be enabled.
- XXX—Don't Care
- R7.5—Reset RST 7.5: If this bit = 1, RST 7.5 flip-flop is reset. This is an additional control to reset RST 7.5.
- MSE—Mask Set Enable: If this bit is high, it enables the functions of bits D₂, D₁, D₀. This is a master control over all the interrupt masking bits. If this bit is low, bits D₂, D₁, and D₀ do not have any effect on the masks.
- M7.5—D₂ = 0, RST 7.5 is enabled.
 = 1, RST 7.5 is masked or disabled.
- M6.5—D₁ = 0, RST 6.5 is enabled.
 = 1, RST 6.5 is masked or disabled.
- M5.5—D₀ = 0, RST 5.5 is enabled.
 = 1, RST 5.5 is masked or disabled.

6. INSTRUCTION EXECUTION AND TIMING DIAGRAM:

Each instruction in 8085 microprocessor consists of two part- operation code (opcode) and operand. The opcode is a command such as ADD and the operand is an object to be operated on, such as a byte or the content of a register.

Instruction Cycle: The time taken by the processor to complete the execution of an instruction. An instruction cycle consists of one to six machine cycles.

Machine Cycle: The time required to complete one operation; accessing either the memory or I/O device. A machine cycle consists of three to six T-states.

T-State: Time corresponding to one clock period. It is the basic unit to calculate execution of instructions or programs in a processor.

To execute a program, 8085 performs various operations as:

- Opcode fetch
- Operand fetch
- Memory read/write
- I/O read/write

External communication functions are:

- Memory read/write
- I/O read/write
- Interrupt request acknowledge

Opcode Fetch Machine Cycle:

It is the first step in the execution of any instruction. The timing diagram of this cycle is given in Fig. 7.

The following points explain the various operations that take place and the signals that are changed during the execution of opcode fetch machine cycle:

T1 clock cycle

- i. The content of PC is placed in the address bus; AD0 - AD7 lines contains lower bit address and A8 – A15 contains higher bit address.
- ii. $\overline{IO/M}$ signal is low indicating that a memory location is being accessed. S1 and S0 also changed to the levels as indicated in Table 1.
- iii. ALE is high, indicates that multiplexed AD0 – AD7 act as lower order bus.

T2 clock cycle

- i. Multiplexed address bus is now changed to data bus.
- ii. The RD signal is made low by the processor. This signal makes the memory device

load the data bus with the contents of the location addressed by the processor.

T3 clock cycle

- i. The opcode available on the data bus is read by the processor and moved to the instruction register.
- ii. The \overline{RD} signal is deactivated by making it logic 1.

T4 clock cycle

- i. The processor decodes the instruction in the instruction register and generate the necessary control signals to execute the instruction. Based on the instruction further operations such as fetching, writing into memory etc takes place.

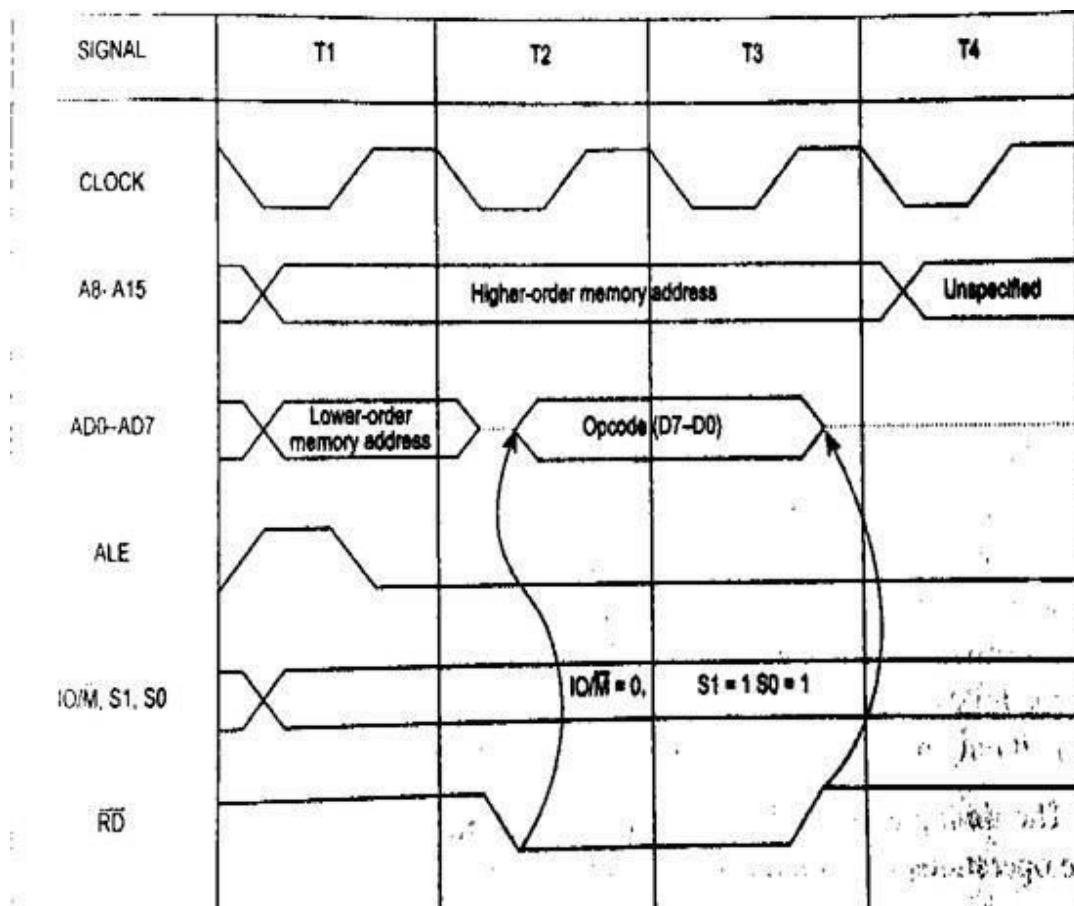


Fig. 7 Timing diagram for opcode fetch cycle

Memory Read Machine Cycle:

The memory read cycle is executed by the processor to read a data byte from memory. The machine cycle is exactly same to opcode fetch except: a) It has three T-states b) The S0 signal is set to 0. The timing diagram of this cycle is given in Fig. 8.

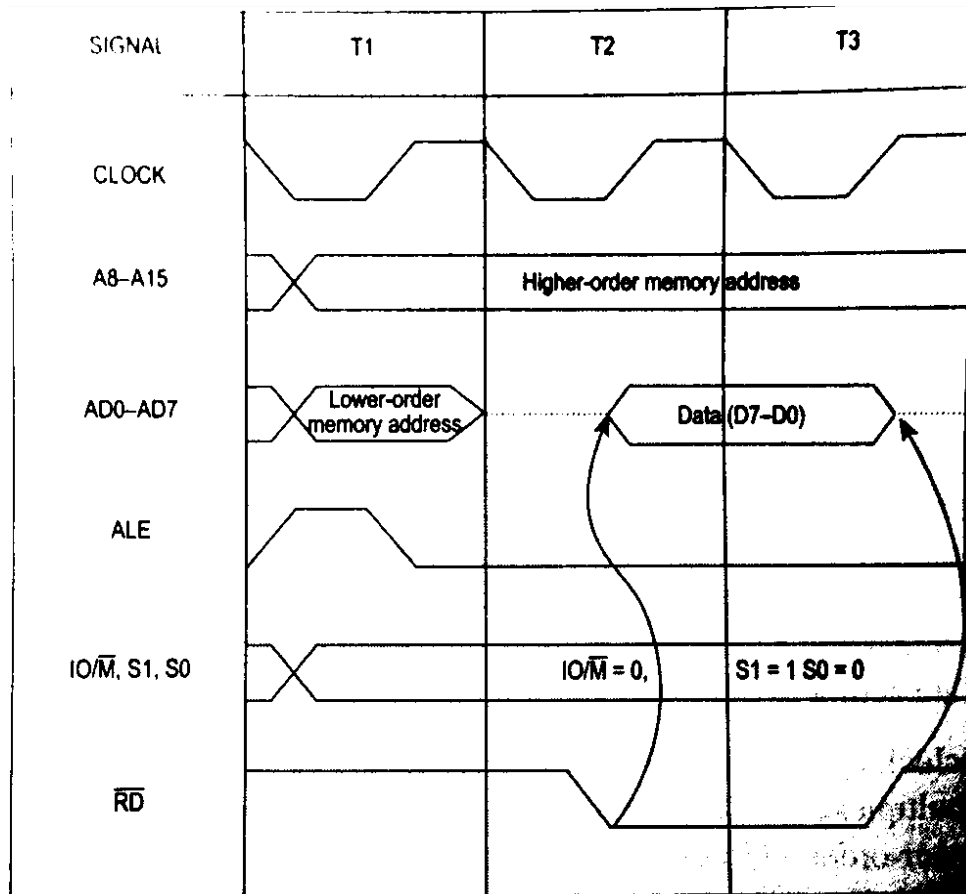


Fig. 8 Timing diagram for memory read machine cycle

Memory Write Machine Cycle:

The memory write cycle is executed by the processor to write a data byte in a memory location. The processor takes three T-states and \overline{WR} signal is made low. The timing diagram of this cycle is given in Fig. 9.

I/O Read Cycle:

The I/O read cycle is executed by the processor to read a data byte from I/O port or from peripheral, which is I/O mapped in the system. The 8-bit port address is placed both in the lower and higher order address bus. The processor takes three T-states to execute this machine cycle. The timing diagram of this cycle is given in Fig. 10.

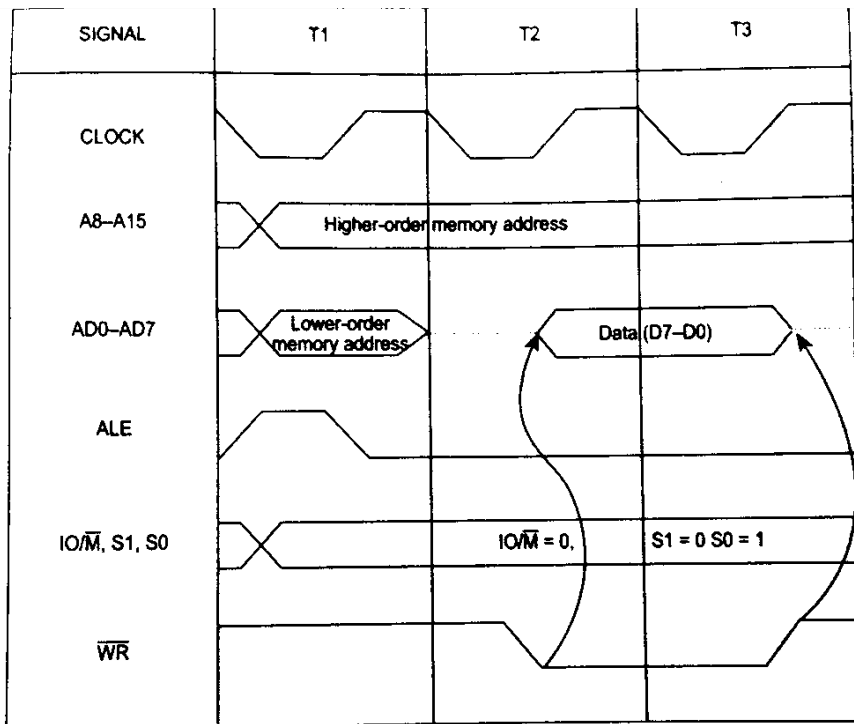


Fig. 9 Timing diagram for memory write machine cycle

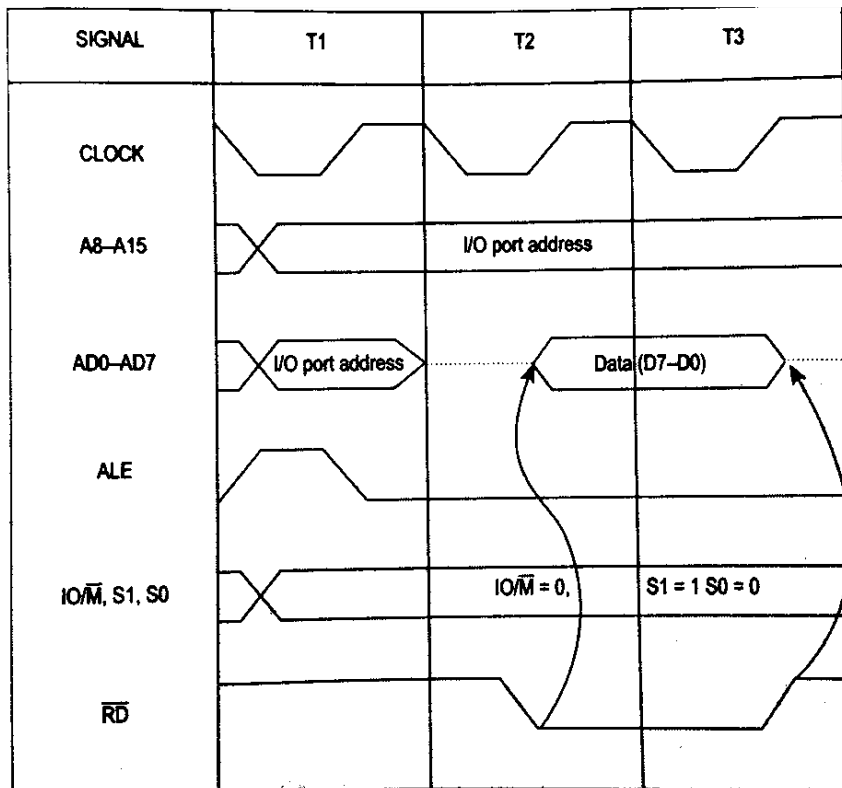


Fig. 10 Timing diagram I/O read machine cycle

I/O Write Cycle:

The I/O write cycle is executed by the processor to write a data byte to I/O port or to a peripheral, which is I/O mapped in the system. The processor takes three T-states to execute this machine cycle. The timing diagram of this cycle is given in Fig. 11.

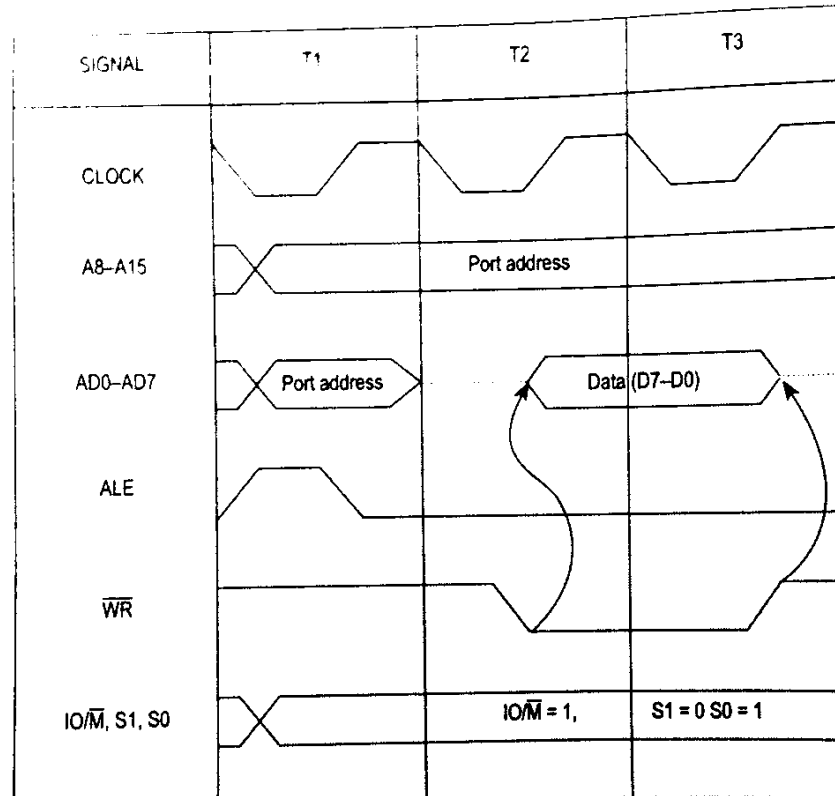


Fig. 11 Timing diagram I/O write machine cycle

Ex: Timing diagram for IN 80H.

The instruction and the corresponding codes and memory locations are given in Table 5.

Table 5 IN instruction

Address	Mnemonics	Opcode
800F	IN 80H	DB
8010		80

- i. During the first machine cycle, the opcode DB is fetched from the memory, placed in the instruction register and decoded.
- ii. During second machine cycle, the port address 80H is read from the next memory location.
- iii. During the third machine cycle, the address 80H is placed in the address bus and the data read from that port address is placed in the accumulator.

The timing diagram is shown in Fig. 12.

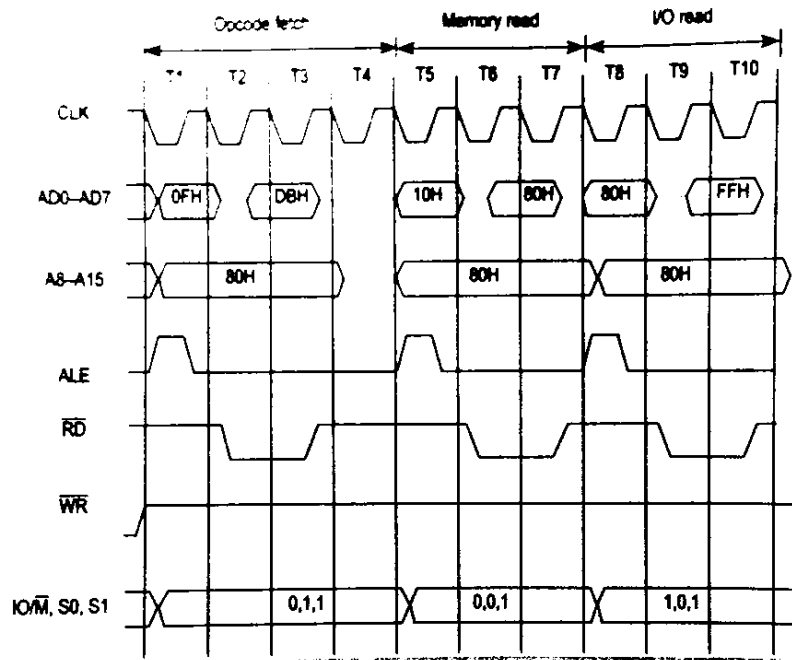


Fig. 12 Timing diagram for the IN instruction

7. 8085 INTERRUPTS

Interrupt Structure:

Interrupt is the mechanism by which the processor is made to transfer control from its current program execution to another program having higher priority. The interrupt signal may be given to the processor by any external peripheral device.

The program or the routine that is executed upon interrupt is called interrupt service routine (ISR). After execution of ISR, the processor must return to the interrupted program. Key features in the interrupt structure of any microprocessor are as follows:

- i. Number and types of interrupt signals available.
- ii. The address of the memory where the ISR is located for a particular interrupt signal. This address is called interrupt vector address (IVA).
- iii. Masking and unmasking feature of the interrupt signals.
- iv. Priority among the interrupts.
- v. Timing of the interrupt signals.
- vi. Handling and storing of information about the interrupt program (status information).

Types of Interrupts:

Interrupts are classified based on their maskability, IVA and source. They are classified as:

- i. Vectored and Non-Vectored Interrupts
 - Vectored interrupts require the IVA to be supplied by the external device that gives the interrupt signal. This technique is vectored, is implemented in number of ways.
 - Non-vectored interrupts have fixed IVA for ISRs of different interrupt signals.
- ii. Maskable and Non-Maskable Interrupts
 - Maskable interrupts are interrupts that can be blocked. Masking can be done by software or hardware means.
 - Non-maskable interrupts are interrupts that are always recognized; the corresponding ISRs are executed.
- iii. Software and Hardware Interrupts
 - Software interrupts are special instructions, after execution transfer the control to predefined ISR.
 - Hardware interrupts are signals given to the processor, for recognition as an interrupt and execution of the corresponding ISR.

Interrupt Handling Procedure:

The following sequence of operations takes place when an interrupt signal is recognized:

- i. Save the PC content and information about current state (flags, registers etc) in the stack.
- ii. Load PC with the beginning address of an ISR and start to execute it.
- iii. Finish ISR when the return instruction is executed.
- iv. Return to the point in the interrupted program where execution was interrupted.

Interrupt Sources and Vector Addresses in 8085:

Software Interrupts:

8085 instruction set includes eight software interrupt instructions called Restart (RST) instructions. These are one byte instructions that make the processor execute a subroutine at predefined locations. Instructions and their vector addresses are given in Table 6.

Table 6 Software interrupts and their vector addresses

Instruction	Machine hex code	Interrupt Vector Address
RST 0	C7	0000H
RST 1	CF	0008H
RST 2	D7	0010H
RST 3	DF	0018H
RST 4	E7	0020H
RST 5	EF	0028H
RST 6	F7	0030H
RST 7	FF	0032H

The software interrupts can be treated as CALL instructions with default call locations. The concept of priority does not apply to software interrupts as they are inserted into the program as instructions by the programmer and executed by the processor when the respective program lines are read.

Hardware Interrupts and Priorities:

8085 have five hardware interrupts – INTR, RST 5.5, RST 6.5, RST 7.5 and TRAP. Their IVA and priorities are given in Table 7.

Table 7 Hardware interrupts of 8085

Interrupt	Interrupt vector address	Maskable or non-maskable	Edge or level triggered	priority
TRAP	0024H	Non-maskable	Level	1
RST 7.5	003CH	Maskable	Rising edge	2
RST 6.5	0034H	Maskable	Level	3
RST 5.5	002CH	Maskable	Level	4
INTR	Decided by hardware	Maskable	Level	5

Masking of Interrupts:

Masking can be done for four hardware interrupts INTR, RST 5.5, RST 6.5, and RST 7.5. The masking of 8085 interrupts is done at different levels. Fig. 13 shows the organization of hardware interrupts in the 8085.

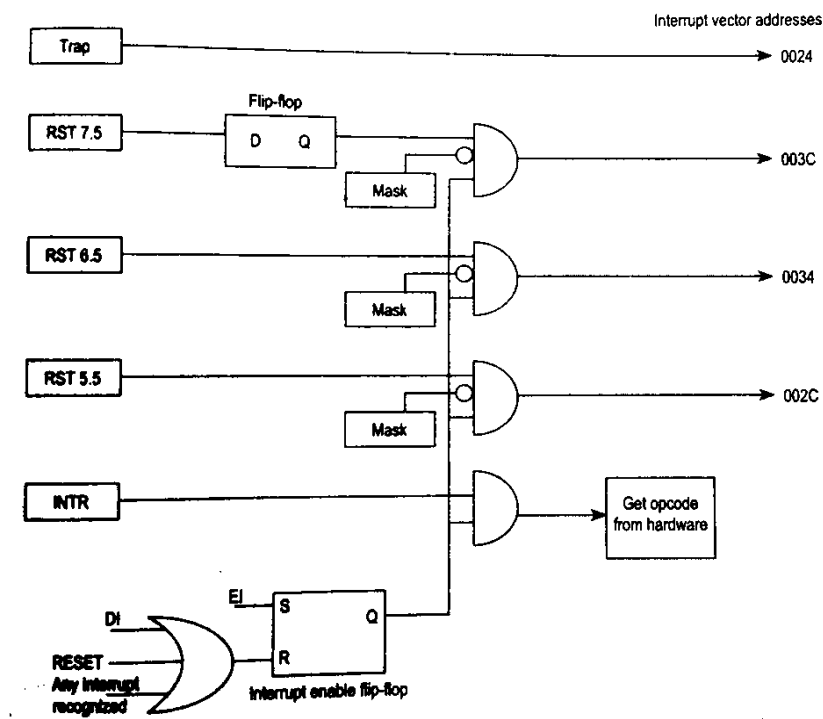


Fig. 13 Interrupt structure of 8085

The Fig. 13 is explained by the following five points:

- i. The maskable interrupts are by default masked by the Reset signal. So no interrupt is recognized by the hardware reset.
- ii. The interrupts can be enabled by the EI instruction.
- iii. The three RST interrupts can be selectively masked by loading the appropriate word in the accumulator and executing SIM instruction. This is called software masking.
- iv. All maskable interrupts are disabled whenever an interrupt is recognized.
- v. All maskable interrupts can be disabled by executing the DI instruction.

RST 7.5 alone has a flip-flop to recognize edge transition. The DI instruction reset interrupt enable flip-flop in the processor and the interrupts are disabled. To enable interrupts, EI instruction has to be executed.

SIM Instruction:

The SIM instruction is used to mask or unmask RST hardware interrupts. When executed, the SIM instruction reads the content of accumulator and accordingly mask or unmask the interrupts. The format of control word to be stored in the accumulator before executing SIM instruction is as shown in Fig. 14.

Bit position	D7	D6	D5	D4	D3	D2	D1	D0
Name	SOD	SDE	X	R7.5	MSE	M7.5	M6.5	M5.5
Explanation	Serial data to be sent	Serial data enable— set to 1 for sending	Not used	Reset RST 7.5 flip-flop	Mask set enable— Set to 1 to mask interrupts	Set to 1 to mask RST 7.5	Set to 1 to mask RST 6.5	Set to 1 to mask RST 5.5

Fig. 14 Accumulator bit pattern for SIM instruction

In addition to masking interrupts, SIM instruction can be used to send serial data on the SOD line of the processor. The data to be send is placed in the MSB bit of the accumulator and the serial data output is enabled by making D6 bit to 1.

RIM Instruction:

RIM instruction is used to read the status of the interrupt mask bits. When RIM instruction is executed, the accumulator is loaded with the current status of the interrupt masks and the pending interrupts. The format and the meaning of the data stored in the accumulator after execution of RIM instruction is shown in Fig. 15.

In addition RIM instruction is also used to read the serial data on the SID pin of the processor. The data on the SID pin is stored in the MSB of the accumulator after the execution of the RIM instruction.

Bit position	D7	D6	D5	D4	D3	D2	D1	D0
Name	SID	I7.5	I6.5	I5.5	IE	M7.5	M6.5	M5.5
Explanation	Serial input data in the SID pin	Set to 1 if RST 7.5 is pending	Set to 1 if RST 6.5 is pending	Set to 1 if RST 5.5 is pending	Set to 1 if interrupts are enabled	Set to 1 if RST 7.5 is masked	Set to 1 if RST 6.5 is masked	Set to 1 if RST 5.5 is masked

Fig. 15 Accumulator bit pattern after execution of RIM instruction

Ex: Write an assembly language program to enables all the interrupts in 8085 after reset.

EI : Enable interrupts

MVI A, 08H : Unmask the interrupts

SIM : Set the mask and unmask using SIM instruction

Timing of Interrupts:

The interrupts are sensed by the processor one cycle before the end of execution of each instruction. An interrupts signal must be applied long enough for it to be recognized. The longest instruction of the 8085 takes 18 clock periods. So, the interrupt signal must be applied for at least 17.5 clock periods. This decides the minimum pulse width for the interrupt signal.

The maximum pulse width for the interrupt signal is decided by the condition that the interrupt signal must not be recognized once again. This is under the control of the programmer.

QUESTIONS:

1. What is the function of a microprocessor in a system?
2. Why is the data bus in 8085 bidirectional?
3. How does microprocessor differentiate between data and instruction?
4. How long would the processor take to execute the instruction LDA 1753H if the T-state duration is $2\mu\text{s}$?
5. Draw the timing diagram of the instruction LDAX B.
6. Sketch and explain the various pins of the 8085.
7. Explain direct addressing mode of 8085 with an example?
8. Draw and explain the timing diagram of the instruction IN 82H.
9. What is meant by 'priority of the interrupts'? Explain the operation of the interrupts structure of the 8085, with the help of a circuit diagram.
10. Explain the bit pattern for SIM instruction. Write the assembly language program lines to enable all the interrupts in the 8085 after reset.
11. Write the logical instructions which affect and which does not affect flags in 8085.
12. Write an ALP in 8085 MPU to reject all the negative readings and add all the positive reading from a set of ten reading stored in memory locations starting at XX60H. When the sum exceeds eight bits produce output FFH to PORT1 to indicate overload otherwise display the sum.
13. Write an ALP in 8085 to eliminate the blanks (bytes with zero value) from a string of eight data bytes. Use two memory pointers: one to get a byte and the other to store the byte.
14. Design an up-down counter to count from 0 to 9 and 9 to 0 continuously with a 1.5 second delay between each count, and display the count at one of the output ports.

UNIT-V INTERFACING AND SUPPORT CHIPS

1. INTERFACING MEMORY AND I/O DEVICES WITH 8085

The programs and data that are executed by the microprocessor have to be stored in ROM/EPROM and RAM, which are basically semiconductor memory chips. The programs and data that are stored in ROM/EPROM are not erased even when power supply to the chip is removed. Hence, they are called non-volatile memory. They can be used to store permanent programs.

In a RAM, stored programs and data are erased when the power supply to the chip is removed. Hence, RAM is called volatile memory. RAM can be used to store programs and data that include, programs written during software development for a microprocessor based system, program written when one is learning assembly language programming and data enter while testing these programs.

Input and output devices, which are interfaced with 8085, are essential in any microprocessor based system. They can be interfaced using two schemes: I/O mapped I/O and memory-mapped I/O. In the I/O mapped I/O scheme, the I/O devices are treated differently from memory. In the memory-mapped I/O scheme, each I/O device is assumed to be a memory location.

2. INTERFACING MEMORY CHIPS WITH 8085

8085 has 16 address lines (A0 - A15), hence a maximum of 64 KB ($= 2^{16}$ bytes) of memory locations can be interfaced with it. The memory address space of the 8085 takes values from 0000H to FFFFH.

The 8085 initiates set of signals such as $\overline{IO/M}$, \overline{RD} and \overline{WR} when it wants to read from and write into memory. Similarly, each memory chip has signals such as \overline{CE} or \overline{CS} (chip enable or chip select), \overline{OE} or \overline{RD} (output enable or read) and \overline{WE} or \overline{WR} (write enable or write) associated with it.

Generation of Control Signals for Memory:

When the 8085 wants to read from and write into memory, it activates $\overline{IO/M}$, \overline{RD} and \overline{WR} signals as shown in Table 8.

Table 8 Status of $\overline{IO/M}$, \overline{RD} and \overline{WR} signals during memory read and write operations

$\overline{IO/M}$	\overline{RD}	\overline{WR}	Operation
0	0	1	8085 reads data from memory
0	1	0	8085 writes data into memory

Using IO/M , RD and WR signals, two control signals MEMR (memory read) and MEMW (memory write) are generated. Fig. 16 shows the circuit used to generate these signals.

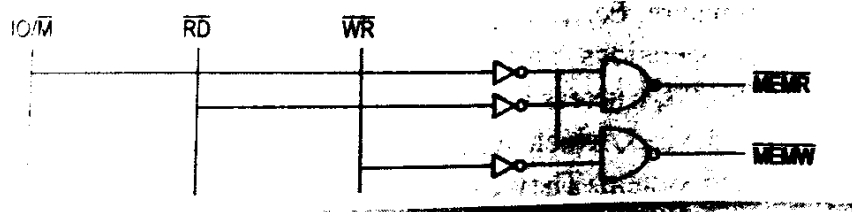


Fig. 16 Circuit used to generate $\overline{\text{MEMR}}$ and $\overline{\text{MEMW}}$ signals

When is $\overline{\text{IO/M}}$ high, both memory control signals are deactivated irrespective of the status of RD and WR signals.

Ex: Interface an IC 2764 with 8085 using NAND gate address decoder such that the address range allocated to the chip is 0000H – 1FFFH.

Specification of IC 2764:

- 8 KB (8×2^{10} byte) EPROM chip
- 13 address lines (2^{13} bytes = 8 KB)

Interfacing:

- 13 address lines of IC are connected to the corresponding address lines of 8085.
- Remaining address lines of 8085 are connected to address decoder formed using logic gates, the output of which is connected to the $\overline{\text{CE}}$ pin of IC.
- Address range allocated to the chip is shown in Table 9.
- Chip is enabled whenever the 8085 places an address allocated to EPROM chip in the address bus. This is shown in Fig. 17.

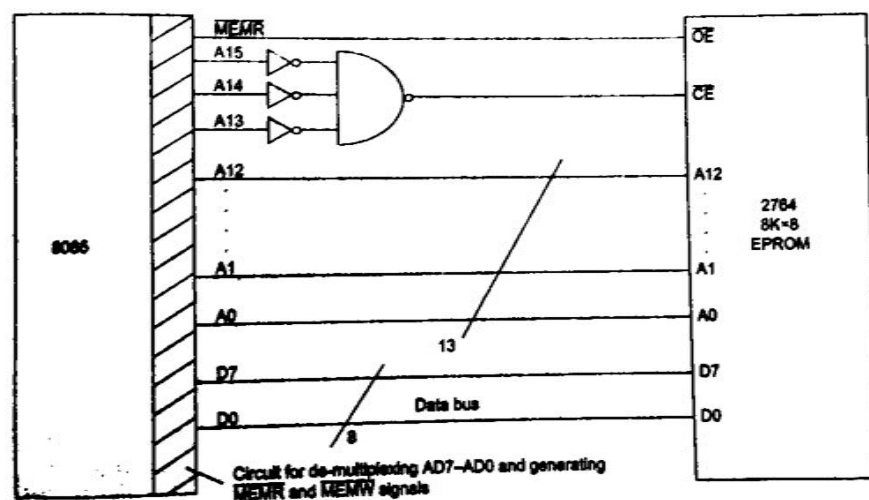


Fig. 17 Interfacing IC 2764 with the 8085

Table 9 Address allocated to IC 2764

A15	A14	A13	A12	A11	A10	A9	A8	A7	A6	A5	A4	A3	A2	A1	A0	Address
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0000H
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0001H
.
.
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1FFEH
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1FFFH

Ex: Interface a 6264 IC (8K x 8 RAM) with the 8085 using NAND gate decoder such that the starting address assigned to the chip is 4000H.

Specification of IC 6264:

- 8K x 8 RAM
- 8 KB = 2^{13} bytes
- 13 address lines

The ending address of the chip is 5FFFH (since $4000H + 1FFFH = 5FFFH$). When the address 4000H to 5FFFH are written in binary form, the values in the lines A15, A14, A13 are 0, 1 and 0 respectively. The NAND gate is designed such that when the lines A15 and A13 carry 0 and A14 carries 1, the output of the NAND gate is 0. The NAND gate output is in turn connected to the $\overline{CE1}$ pin of the RAM chip. A NAND output of 0 selects the RAM chip for read or write operation, since CE2 is already 1 because of its connection to +5V. Fig. 18 shows the interfacing of IC 6264 with the 8085.

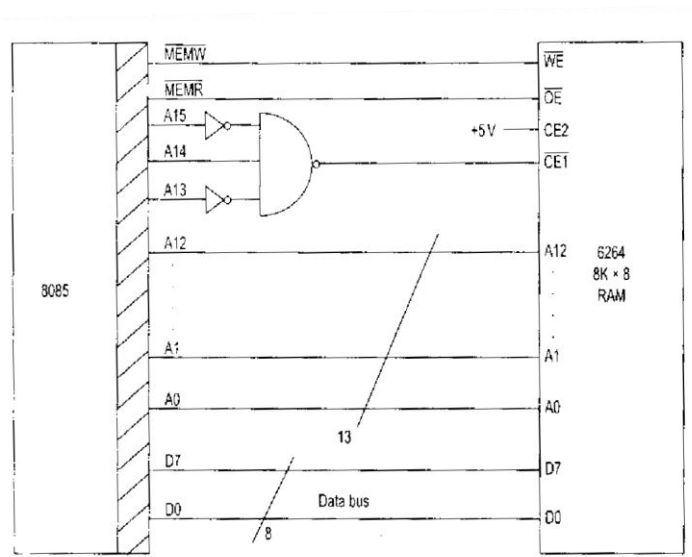


Fig. 18 Interfacing 6264 IC with the 8085

Ex: Interface two 6116 ICs with the 8085 using 74LS138 decoder such that the starting addresses assigned to them are 8000H and 9000H, respectively.

Specification of IC 6116:

- 2 K x 8 RAM
- 2 KB = 2^{11} bytes
- 11 address lines

6116 has 11 address lines and since 2 KB, therefore ending addresses of 6116 chip 1 is and chip 2 are 87FFH and 97FFH, respectively. Table 10 shows the address range of the two chips.

Table 10 Address range for IC 6116

A15	A14	A13	A12	A11	A10	A9	A8	A7	A6	A5	A4	A3	A2	A1	A0	Address
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8000H
.
1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	87FFH (RAM chip 1)
1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	9000H
.
1	0	0	1	0	1	1	1	1	1	1	1	1	1	1	1	97FFH (RAM chip 2)

Interfacing:

- Fig. 19 shows the interfacing.
- A0 – A10 lines of 8085 are connected to 11 address lines of the RAM chips.
- Three address lines of 8085 having specific value for a particular RAM are connected to the three select inputs (C, B and A) of 74LS138 decoder.
- Table 10 shows that A13=A12=A11=0 for the address assigned to RAM 1 and A13=0, A12=1 and A11=0 for the address assigned to RAM 2.
- Remaining lines of 8085 which are constant for the address range assigned to the two RAM are connected to the enable inputs of decoder.
- When 8085 places any address between 8000H and 87FFH in the address bus, the select inputs C, B and A of the decoder are all 0. The Y0 output of the decoder is also 0, selecting RAM 1.
- When 8085 places any address between 9000H and 97FFH in the address bus, the select inputs C, B and A of the decoder are 0, 1 and 0. The Y2 output of the decoder is also 0, selecting RAM 2.

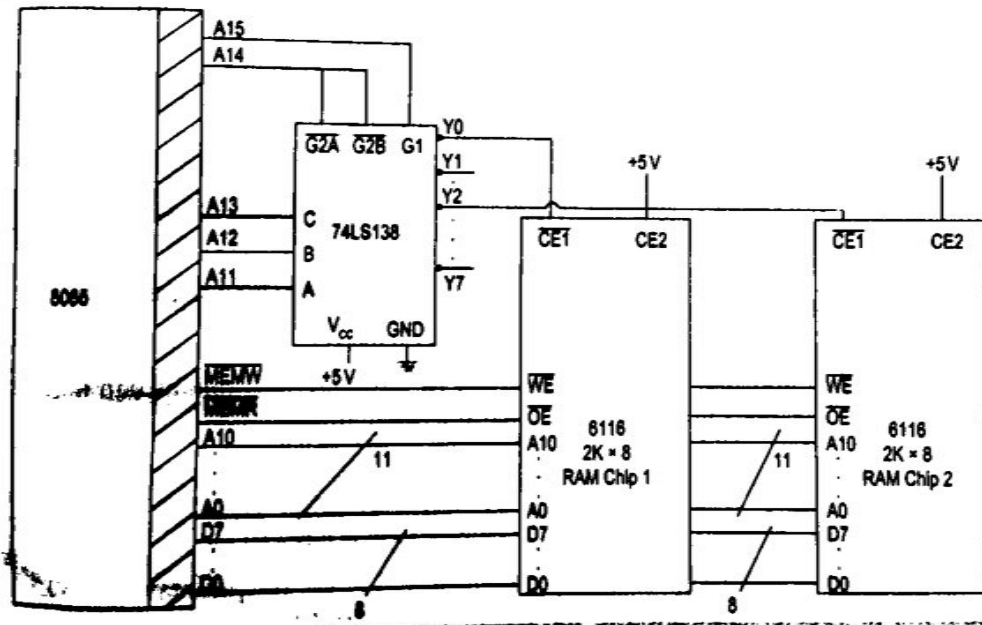


Fig. 19 Interfacing two 6116 RAM chips using 74LS138 decoder

3. PERIPHERAL MAPPED I/O INTERFACING

In this method, the I/O devices are treated differently from memory chips. The control signals I/O read (\overline{IOR}) and I/O write (\overline{IOW}), which are derived from the $\overline{IO/M}$, \overline{RD} and \overline{WR} signals of the 8085, are used to activate input and output devices, respectively. Generation of these control signals is shown in Fig. 20. Table 11 shows the status of $\overline{IO/M}$, \overline{RD} and \overline{WR} signals during I/O read and I/O write operation.

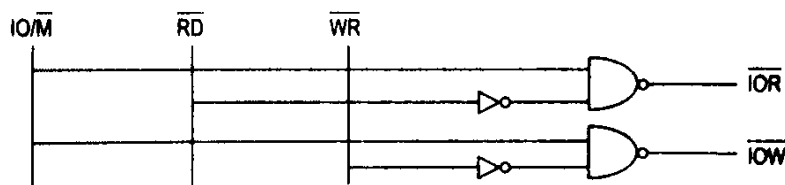


Fig. 20 Generation of \overline{IOR} and \overline{IOW} signals

IN instruction is used to access input device and OUT instruction is used to access output device. Each I/O device is identified by a unique 8-bit address assigned to it. Since the control signals used to access input and output devices are different, and all I/O device use 8-bit address, a maximum of 256 (2^8) input devices and 256 output devices can be interfaced with 8085.

Table 11 Status of \overline{IOR} and \overline{IOW} signals in 8085.

$\overline{IO/M}$	\overline{RD}	\overline{WR}	\overline{IOR}	\overline{IOW}	Operation
1	0	1	0	1	I/O read operation

1	1	0	1	0	I/O write operation
0	X	X	1	1	Memory read or write operation

Ex: Interface an 8-bit DIP switch with the 8085 such that the address assigned to the DIP switch is F0H.

IN instruction is used to get data from DIP switch and store it in accumulator. Steps involved in the execution of this instruction are:

- i. Address F0H is placed in the lines A0 – A7 and a copy of it in lines A8 – A15.
- ii. The $\overline{\text{IOR}}$ signal is activated ($\overline{\text{IOR}} = 0$), which makes the selected input device to place its data in the data bus.
- iii. The data in the data bus is read and store in the accumulator.

Fig. 21 shows the interfacing of DIP switch.

A7	A6	A5	A4	A3	A2	A1	A0	= F0H
1	1	1	1	0	0	0	0	

A0 – A7 lines are connected to a NAND gate decoder such that the output of NAND gate is 0. The output of NAND gate is ORed with the $\overline{\text{IOR}}$ signal and the output of OR gate is connected to $\overline{\text{IG}}$ and $\overline{\text{2G}}$ of the 74LS244. When 74LS244 is enabled, data from the DIP switch is placed on the data bus of the 8085. The 8085 read data and store in the accumulator. Thus data from DIP switch is transferred to the accumulator.

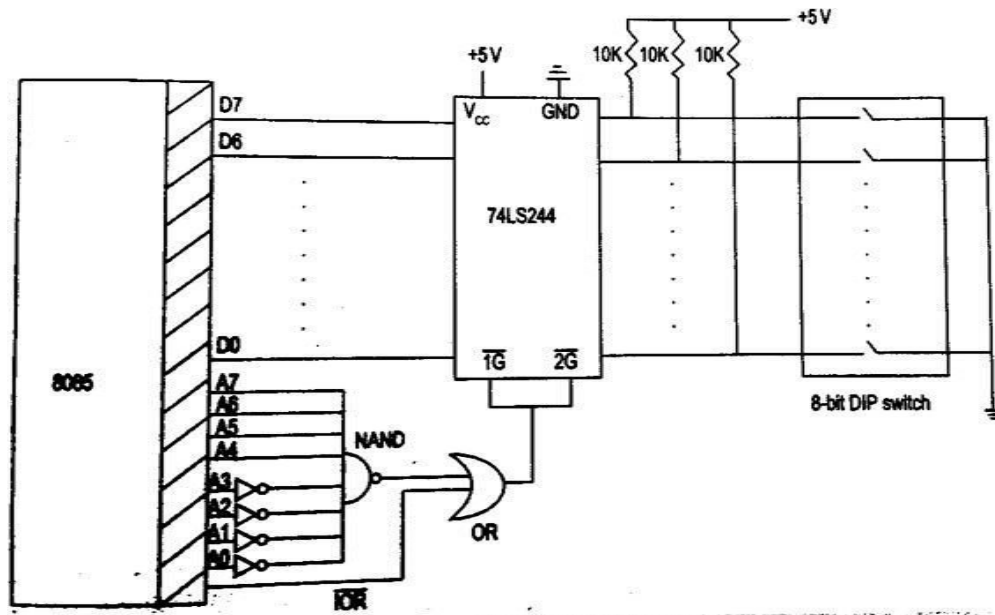


Fig. 21 interfacing of 8-bit DIP switch with 8085

4. MEMORY MAPPED I/O INTERFACING

In memory-mapped I/O, each input or output device is treated as if it is a memory location. The $\overline{\text{MEMR}}$ and $\overline{\text{MEMW}}$ control signals are used to activate the devices. Each input or output device is identified by unique 16-bit address, similar to 16-bit address assigned to

memory location. All memory related instruction like LDA 2000H, LDAX B, MOV A, M can be used.

Since the I/O devices use some of the memory address space of 8085, the maximum memory capacity is lesser than 64 KB in this method.

Ex: Interface an 8-bit DIP switch with the 8085 using logic gates such that the address assigned to it is F0F0H.

Since a 16-bit address has to be assigned to a DIP switch, the memory-mapped I/O technique must be used. Using LDA F0F0H instruction, the data from the 8-bit DIP switch can be transferred to the accumulator. The steps involved are:

- i. The address F0F0H is placed in the address bus A0 – A15.
- ii. The MEMR signal is made low for some time.
- iii. The data in the data bus is read and stored in the accumulator.

Fig. 22 shows the interfacing diagram.

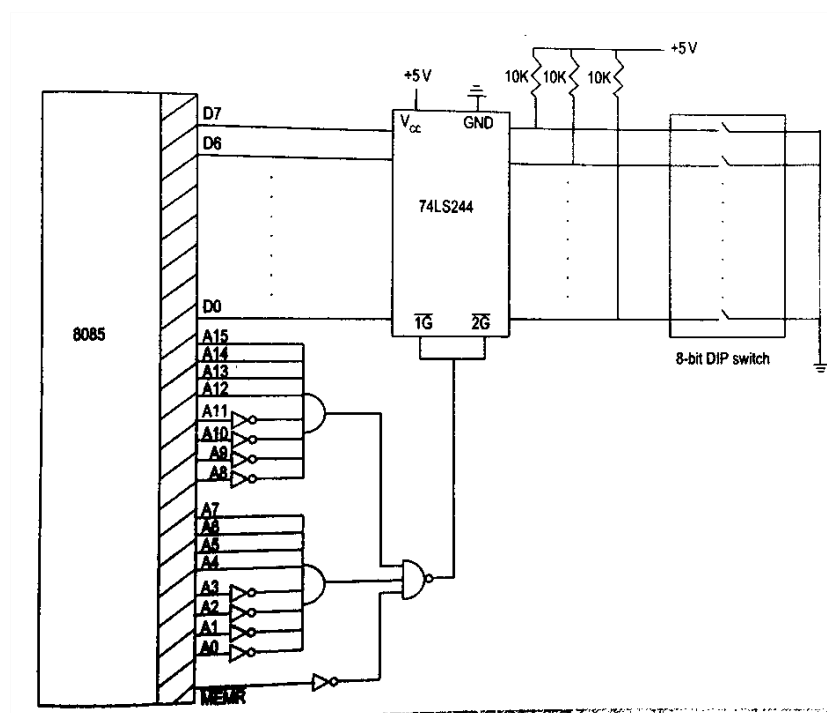


Fig. 22 Interfacing 8-bit DIP switch with 8085

When 8085 executes the instruction LDA F0F0H, it places the address F0F0H in the address lines A0 – A15 as:

A15	A14	A13	A12	A11	A10	A9	A8	A7	A6	A5	A4	A3	A2	A1	A0	
1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	= F0F0H

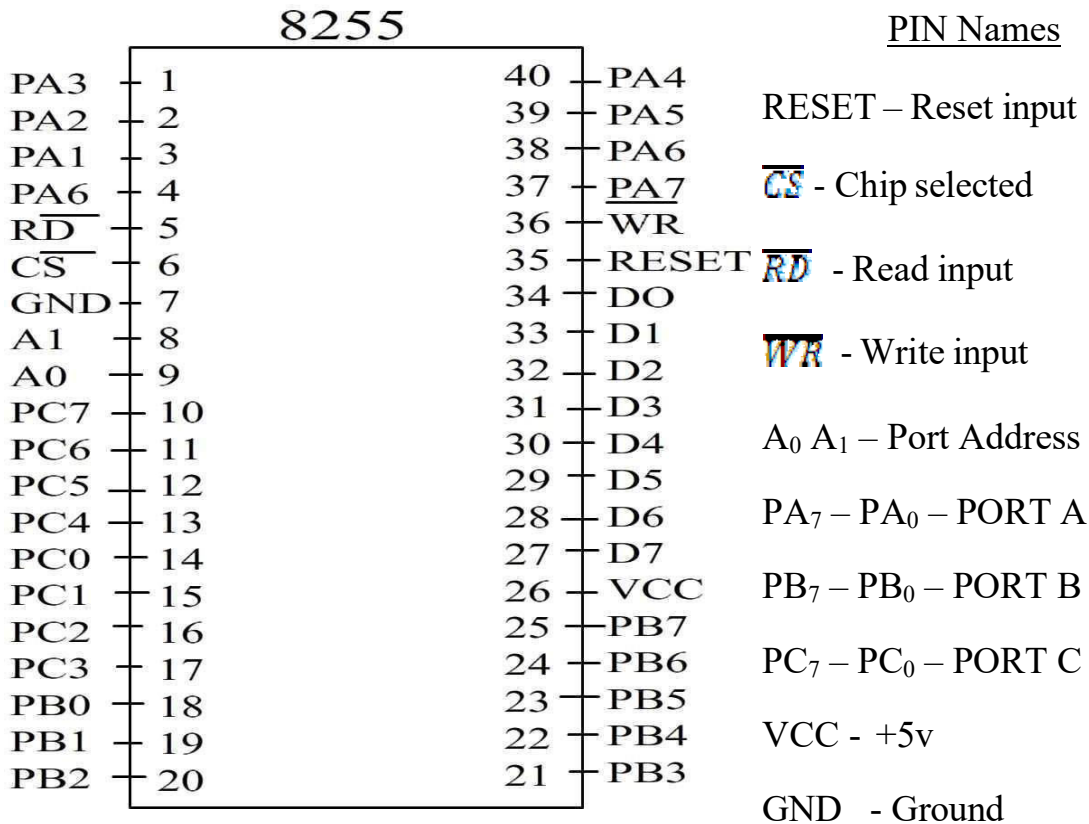
The address lines are connected to AND gates. The output of these gates along with MEMR signal are connected to a NAND gate, so that when the address F0F0H is placed in the address bus and MEMR = 0 its output becomes 0, thereby enabling the buffer 74LS244. The data from the DIP switch is placed in the 8085 data bus. The 8085 reads the data from

the data bus and stores it in the accumulator.

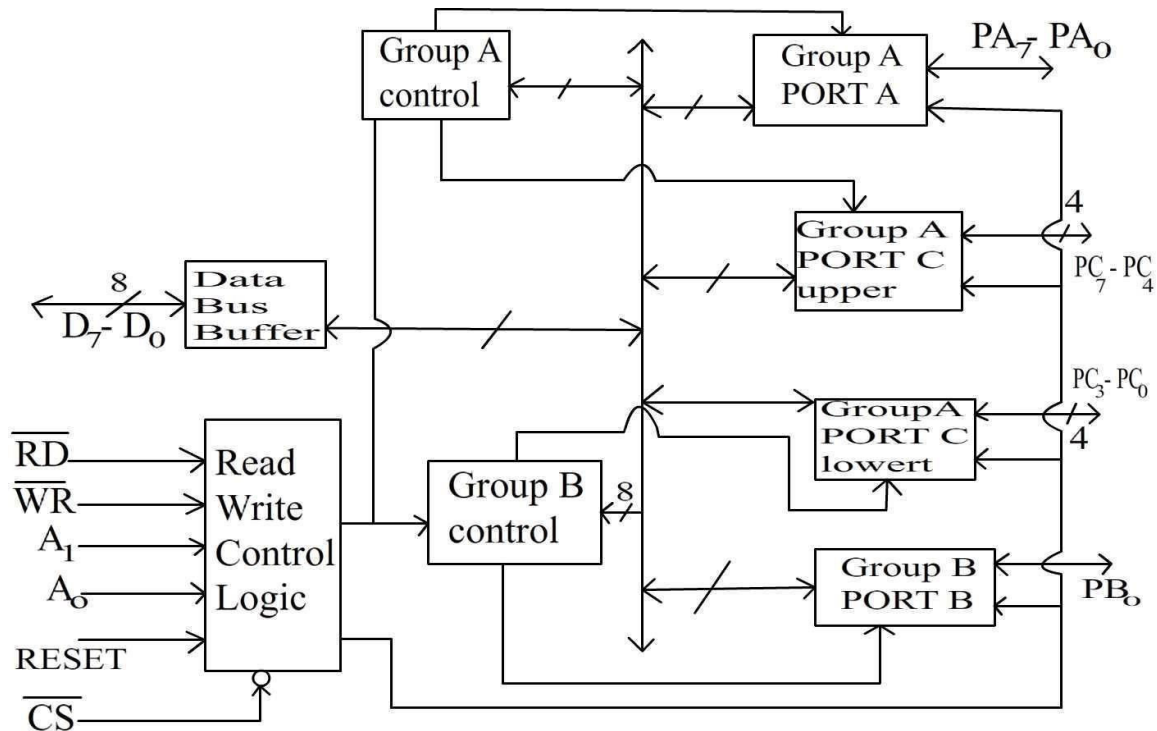
INTEL 8255: (Programmable Peripheral Interface)

The 8255A is a general purpose programmable I/O device designed for use with Intel microprocessors. It consists of three 8-bit bidirectional I/O ports (24 I/O lines) that can be configured to meet different system I/O needs. The three ports are PORT A, PORT B & PORT C. Port A contains one 8-bit output latch/buffer and one 8-bit input buffer. Port B is same as PORT A or PORT B. However, PORT C can be split into two parts PORT C lower (PC₀-PC₃) and PORT C upper (PC₇-PC₄) by the control word. The three ports are divided in two groups Group A (PORT A and upper PORT C) Group B (PORT B and lower PORT C). The two groups can be programmed in three different modes. In the first mode (mode 0), each group may be programmed in either input mode or output mode (PORT A, PORT B, PORT C lower, PORT C upper). In mode 1, the second's mode, each group may be programmed to have 8-lines of input or output (PORT A or PORT B) of the remaining 4-lines (PORT C lower or PORT C upper) 3-lines are used for hand shaking and interrupt control signals. The third mode of operation (mode 2) is a bidirectional bus mode which uses 8-line (PORT A only for a bidirectional bus and five lines (PORT C upper 4 lines and borrowing one from other group) for handshaking.

The 8255 is contained in a 40-pin package, whose pin out is shown below:



The block diagram is shown below:



Functional Description:

This support chip is a general purpose I/O component to interface peripheral equipment to the microcomputer system bus. It is programmed by the system software so that normally no external logic is necessary to interface peripheral devices or structures.

Data Bus Buffer:

It is a tri-state 8-bit buffer used to interface the chip to the system data bus. Data is transmitted or received by the buffer upon execution of input or output instructions by the CPU. Control words and status information are also transferred through the data bus buffer. The data lines are connected to BDB of p.

Read/Write and logic control:

The function of this block is to control the internal operation of the device and to control the transfer of data and control or status words. It accepts inputs from the CPU address and control buses and in turn issues command to both the control groups.

\overline{CS} Chip Select:

A low on this input selects the chip and enables the communication between the 8255 A & the CPU. It is connected to the output of address decode circuitry to select the device when it \overline{RD} (Read). A low on this input enables the 8255 to send the data or status information to the CPU on the data bus.

WR (Write):

A low on this input pin enables the CPU to write data or control words into the 8255 A.

A₁, A₀ port select:

These input signals, in conjunction with the **RD** and **WR** inputs, control the selection of one of the three ports or the control word registers. They are normally connected to the least significant bits of the address bus (A₀ and A₁).

Following Table gives the basic operation,

A ₁	A ₀	<u>RD</u>	<u>WR</u>	<u>CS</u>	Input operation
0	0	0	1	0	PORT A → Data bus
0	1	0	1	0	PORT B → Data bus
1	0	0	1	0	PORT C → Data bus
					<u>Output operation</u>
0	0	1	0	0	Data bus → PORT A
0	1	1	0	0	Data bus → PORT B
1	0	1	0	0	Data bus → PORT C
1	1	1	0	0	Data bus → control

All other states put data bus into tri-state/illegal condition.

RESET:

A high on this input pin clears the control register and all ports (A, B & C) are initialized to input mode. This is connected to RESET OUT of 8255. This is done to prevent destruction of circuitry connected to port lines. If port lines are initialized as output after a power up or

reset, the port might try to output into the output of a device connected to same inputs might destroy one or both of them.

PORTs A, B and C:

The 8255A contains three 8-bit ports (A, B and C). All can be configured in a variety of functional characteristic by the system software.

PORTA:

One 8-bit data output latch/buffer and one 8-bit data input latch.

PORT B:

One 8-bit data output latch/buffer and one 8-bit data input buffer.

PORT C:

One 8-bit data output latch/buffer and one 8-bit data input buffer (no latch for input). This port can be divided into two 4-bit ports under the mode control. Each 4-bit port contains a 4-bit latch and it can be used for the control signal outputs and status signals inputs in conjunction with ports A and B.

Group A & Group B control:

The functional configuration of each port is programmed by the system software. The control words outputted by the CPU configure the associated ports of the each of the two groups. Each control block accepts command from Read/Write content logic receives controlwords from the internal data bus and issues proper commands to its associated ports.

Control Group A – Port A & Port C upper

Control Group B – Port B & Port C lower

The control word register can only be written into No read operation if the control word register is allowed.

Operation Description:

Mode selection:

There are three basic modes of operation that can be selected by the system software.

Mode 0: Basic Input/output

Mode 1: Strobes Input/output

Mode 2: Bi-direction bus.

When the reset input goes HIGH all ports are set to mode '0' as input which means all 24 lines are in high impedance state and can be used as normal input. After the reset is removed the 8255A remains in the input mode with no additional initialization. During the execution of the program any of the other modes may be selected using a single output instruction.

The modes for PORT A & PORT B can be separately defined, while PORT C is divided into two portions as required by the PORT A and PORT B definitions. The ports are thus divided into two groups Group A & Group B. All the output register, including the status flip-flop will be reset whenever the mode is changed. Modes of the two groups may be combined for any desired I/O operation e.g. Group A in mode '1' and group B in mode '0'.

The basic mode definitions with bus interface and the mode definition format are given in fig (a) & (b),

